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APPARATUS FOR STUDY OF THE BREAKUP OF LIQUID DROPS BY HIGH VELOCITY AIRSTREAMS

BY

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ABSTRACT

The use of a blast gun and a shock tube for investigating the breakup of liquid drops, 3–4 mm. in diameter, in high velocity airstreams is described. The flow in the shock tube was found to be uniform and predictable, and the pressures and velocities behind the shock front could be regulated to whatever was desired. The preliminary observations were made at an airstream velocity of approximately Mach 1, and a pressure of 1 atmosphere behind the shock front. The photographs show a gradual transition in the breakup mechanism as the forces caused by the high velocity airstream acting on the drop are increased. The "bursting bag" mode is overcome by the violence of the forces of the high velocity airstream and the mechanism of "surface stripping" becomes predominant.

INTRODUCTION

The manner in which liquid masses in an airstream divide into drops, and the ultimate drop size distribution of the resulting aerosol are of prime importance in the field of atomization. The energies required to reduce liquids to small particles are very large. In order to study this high energy process, high energy methods were required.

The present investigation is being conducted on the basic factors involved in the breakup of liquids and the formation of aerosols. The investigators limited this study to the breakup mechanism of liquid drops by high velocity airstreams. Initially a blast gun was used, and for reasons discussed, a shock tube was used in the later phases.

Previous investigations of factors involved in high velocity breakup of liquids include the studies of drop breakup by Engel (1),[3] Hansen

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[3] The boldface numbers in parentheses refer to the references appended to this paper.

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and Domich (2) and Lane (3) and the breakup of drops and sprays by Priem (4). The apparatus described in this study differs from those of previous investigators in that both the pressure and velocity of the airstream following the shock front were controlled, and that the drop was suspended in such a manner that no abnormal external physical forces were acting upon it in such a way as to contribute to or interfere with its breakup. These factors will be discussed individually later in this report.

When a liquid drop is subjected to an airstream it becomes deformed until the internal pressure arising from the surface tension balances the external air pressure distribution (5). If the relative velocity is high enough to overcome the internal forces, the drop becomes unstable and breaks up. The mechanism of breakup depends on how greatly the velocity exceeds the minimum velocity required to produce instability.

An experimental means to produce high velocity airstreams for drop testing was sought. Velocities of the order of Mach 1.0 and higher were needed. An ordinary steady-flow air nozzle is not suitable because an initially spherical drop cannot be formed within the jet. Release of a drop into a horizontal air jet has been used by others (6), but the drop impinging on the edge of the air jet receives a glancing blow and is not hit by an airstream of the desired velocity.

BLAST GUN STUDIES

A variation of the technique originated by Lane and co-workers (3), using a blast gun was first studied. The blast gun is essentially a pressurized chamber separated from an open-ended expansion chamber by a frangible diaphragm. At high pressures, the bursting diaphragm allows a shock wave and an air blast to form within the expansion chamber and to form an air jet directed onto the externally-located test object. Figure 1 is a diagram of the blast gun and its related equipment.

The blast gun was constructed of 1-in. steel tubing. The section to the left of the diaphragm or compression chamber can be pressurized from a nitrogen cylinder to a maximum pressure of 1500 psi. The membrane diaphragm material and thickness were selected so that the bursting strength would be slightly above the pressures prevailing at the operating conditions. The open-ended section, or expansion chamber, remains at atmospheric pressure until the blast arrives. The blast gun is fired by energizing a dagger-tipped solenoid located in the compression chamber. The dagger ruptures the diaphragm, releasing the shock down the expansion chamber. The air blast emerges at high velocity from the open-ended chamber into the observation field and subsequently onto the test object (a liquid drop).

DROP SUSPENSION

It was felt, for this investigation, that suspending the drop on cross-hairs might possibly distort it and hinder the breakup of the drop when struck by the air blast. Consequently, an air-suspension column similar to that of Blanchard (7) was developed.

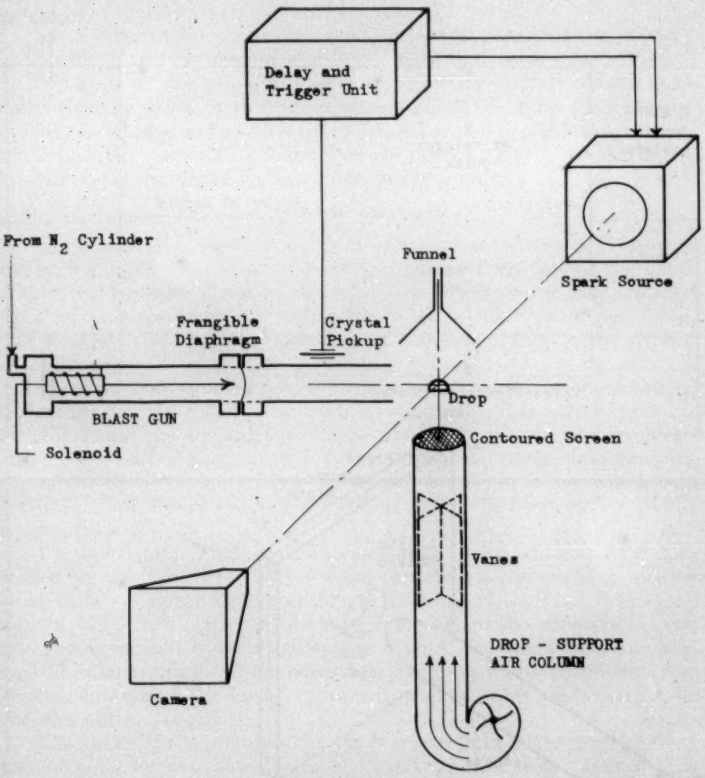


FIG. 1. Diagram of blast gun.

As shown in Fig. 1, the air-suspension column for support of the drop is a vertically-aligned, 2-in.-diameter tube which was placed below the test area. This tube can be supplied with a low-velocity air flow from a variable-speed blower. Two perpendicular 6-in. vanes within

the tube serve to straighten the flow which passes through a screen situated at its upper end. The screen, about 50 mesh, is contoured so that its flow resistance varies, uniformly decreasing from the center outward. By careful adjustment of these contours, the screen serves to reverse the normal parabolic velocity gradient. A funnel placed 5 in.

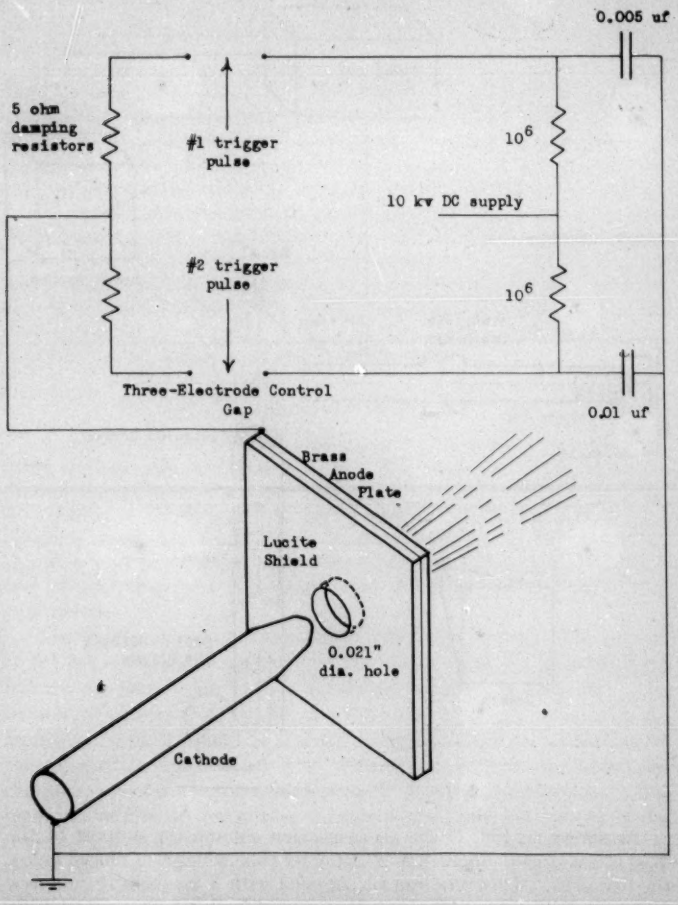


FIG. 2. Basic components of flash circuit.

above this screen further stabilizes the flow. A drop placed in the center of the air column opposite the muzzle of the blast gun (expansion chamber end) remains balanced in this position until evaporated, without support other than that from the air column, but becomes flattened on its lower surface and appears to rotate about its vertical axis of symmetry. The air-column velocity is then lowered to maintain balance of the drop during evaporation.

Later experiments showed that the entire test field could be enclosed in a cubic chamber, 6 in. or a side, for light-tight photographic work. In this type of operation, the blast-gun muzzle and air-column tube enter through holes in the side and bottom of the cube, respectively. A film carrier is attached directly to a side parallel to the gun and tube, with the spark source in a channel on the opposite side. The lid of the box serves as the upper baffle, replacing the funnel.

PHOTOGRAPHIC OBSERVATION

Drop breakup caused by the air blast was recorded as a shadowgraph by back illumination from a spark gap placed 5 ft. behind the drop. The subject-to-film distance was kept to a minimum of 4 in., so that the drop was photographed only slightly larger than actual size. Figure 1 shows the relative placement of this equipment in the experimental setup.

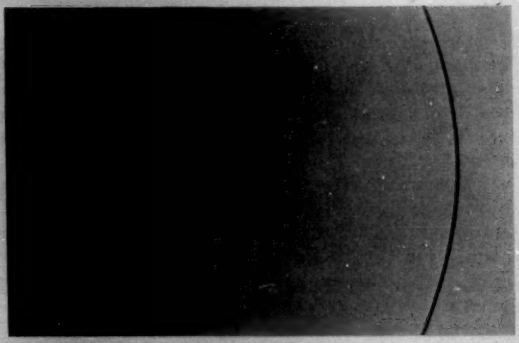
A crystal microphone taped to the gun barrel senses the shock caused by triggering the blast gun with a solenoid. This shock pulse is amplified and fed to hydrogen-thyratron delay and spark-trigger circuits. For double shadowgraph observations, the trigger unit can produce two high-voltage output pulses adjustable in delay which trip separate control gaps in the spark circuit (**8**). The basic circuit layout is given in Fig. 2.

The spark unit contains two capacitors (0.005 μf. and 0.010 μf., respectively) charged to 10 kv. Each capacitor is connected to its own three-electrode control gap and then to a common Liebessart-type spark gap (**8**, **9**). Each capacitor discharges only when the appropriate control gap is ionized by a trigger pulse. Thus, the spark unit provides two illumination flashes from a common spark gap and superimposes two shadow images of the event (obtained after suitable time delays) on the film plate.

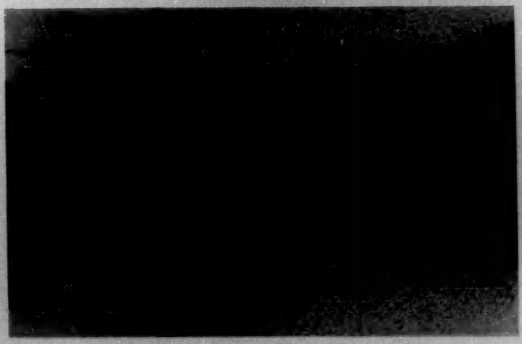
The anode of the illumination gap is a $\frac{1}{8}$-in. brass plate shielded from the cathode by a Lucite coating. A no. 75 (0.021-in.) hole drilled through this assembly is aligned on the axis of a round-pointed cathode rod. The discharge is guided from the cathode into the anode hole by the insulating coating of Lucite so that light is emitted from the 0.021-in. hole. This approximation to a point source gives sufficiently sharp negatives for the drop breakup pictures.

This spark circuit is a modification of one reported (**8**) to have an

(*A*) Single flash showing drop, shock front, and shock reflection.



(*B*) Double flash showing shock front, air blast, stripping from the surface of the drop, and turbulence.



(*C*) Double flash showing faint image of droplet cloud, displaced cloud downstream, and turbulence.

FIG. 3. Shadowgraphs of drop breakup by blast gun.

effective photographic duration of $\frac{1}{8}$ μs. Use of the shadowgraph technique allows recording of shock fronts and flow patterns around the test drops. Slight air pressure variations accompanied by changes in refractive index in the disturbances caused refraction of light rays passing through the flow field. Such pressure changes are shown by the darkened and lightened bands on the film plates (Fig. 3).

FIG. 4. First stage of radial acceleration of ligaments from a drop. (3× enlargement, reduced $\frac{1}{3}$ in reproduction.)

DROP BREAKUP BY THE BLAST GUN TECHNIQUE

After construction, this gun was hydrostatically pressure-tested to 1500 psi. A number of trials were made to determine the most suitable diaphragm material. Sheet copper, aluminum, steel, and a variety of papers and plastics were tested. A compression chamber pressure of 350 psi. was selected for these tests, using two thicknesses of10-mil acetate-base film for the diaphragm. The air test velocity was 1290 ft./sec.

Figure 3 consists of a series of shadowgraphs illustrating the breakup of a 4-mm. drop of diethylene glycol monobutyl ether. The scale in these shadowgraphs is indicated by the blast gun barrel which is 1.5 in. in outside diameter. The air-blast issues from the blast gun barrel which is represented by the block rectangle on the left of the shadowgraphs. In Fig. 3*A*, a single-flash picture, the shock front has just passed over the drop. The shock reflection from the drop is clearly shown, and it appears as though a portion of the downstream side of the drop is breaking loose.

Figure 3*B* is a double-flash superimposed picture in which the first exposure was taken after the spherically-expanding shock front had passed the right hand side of the picture. During this interval, some stripping of fine droplets into the trailing cone occurred, although the detached or reflected shock wave which had formed upstream indicates appreciable mass still remaining in the original drop. The mottled region downstream from this cone represents the turbulent drop wake.

Figure 3*C* shows a much later stage in the shatter of a drop. This double-flash shadowgraph shows a faint first stage image of the droplet cloud, and later, a stronger one displaced downstream about $\frac{1}{2}$ in. on the photograph, as the material is accelerated in the jet. Although a large cloud has formed, the bulk of the material is lagging in relatively large discrete droplet form.

Figure 4 is an enlarged (3X) double-flash picture, showing in the first stage the radial acceleration of ligaments from a drop. This radial acceleration of the ligaments is undoubtedly due to the centrifugal action of the jet vortex.

Preliminary tests with the blast gun indicated that a shock tube would be necessary for the control and accuracy required for the following reasons: The air velocity at the point of breakup with the blast gun was uncertain since the air velocity decreased with distance away from the end of the blast tube. Control of the test conditions was also made difficult by air pressures in excess of one atmosphere at the point of breakup. In addition, the jet vortex from the blast gun and the turbulent radial acceleration of the ligaments stripped from the drop added to the inaccuracy of the experiments (Figs. 3 and 4).

SHOCK TUBE STUDIES

A shock tube is a versatile instrument for producing a uniform gas flow under controlled conditions. The flow in the shock tube is uniform and predictable, which is not the case with the blast gun. The pressure behind the shock front in the shock tube can be regulated to whatever operating pressure is desired for the test, whereas such control could not be obtained when using the blast gun. The two basic differences between the blast gun and the shock tube are (1) that the tube end is closed on the shock tube rather than open as in the blast gun allowing pressure adjustment within the expansion chamber, and (2) that the test area is enclosed within the shock tube rather than being externally located as in the blast gun.

SHOCK TUBE MECHANICS

The shock tube is extensively used in fluid mechanics research. The theoretical treatment and experimental verifications of shock tubes are described in the literature (**10, 11, 12**). Therefore, only a brief account of their operation will be presented here.

In a tube of uniform cross section, the sudden removal of a barrier (a frangible diaphragm), across which there is a sufficient pressure differential, will give rise to a shock wave propagated down the low-pressure section. A shock front is propagated at a speed always in the excess of the local sonic velocity. It is followed by a zone of uniform gas flow, representing gas originally in the expansion chamber which has been compressed and accelerated by passage of the shock front. The particle velocity of this flow, as distinguished from the front velocity, may or may not be supersonic, depending on the initial conditions.

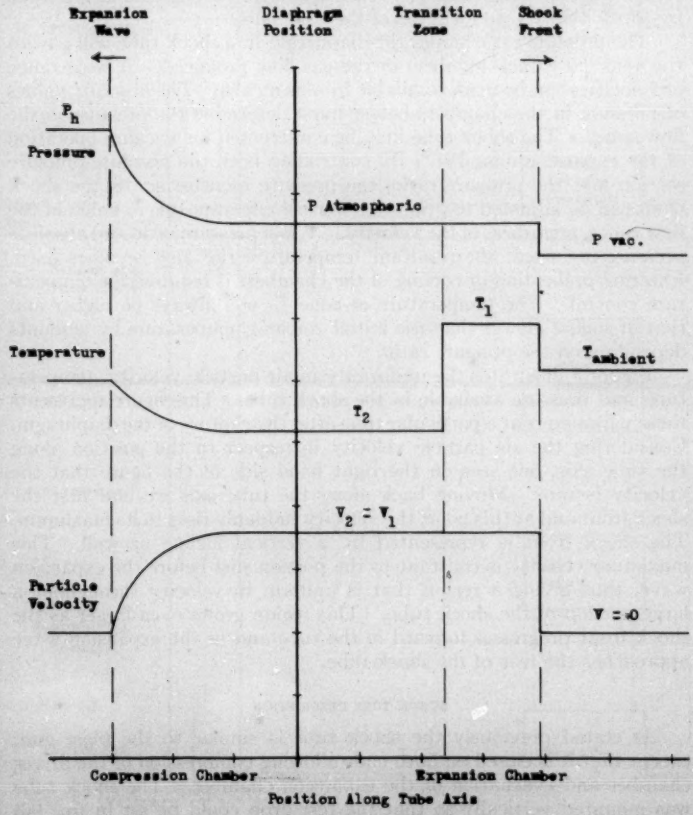


FIG. 5. Shock tube pressure, temperature, and velocity profiles.

Following this first flow zone is a second zone consisting of gas expanding from the compression chamber. Both zones are traveling with a common velocity and pressure but are separated by a surface of discontinuity. Temperature, density and entropy of the regions differ, varying discontinuously across the separation surface (the transition zone of Fig. 5).

Thus, two steady-flow zones are available from a shock tube. These portions of the blast persist until met or overtaken by reflections of the shock front or rarefaction waves from the end caps. Hence, the duration of test flow undisturbed by reflections from the ends can be governed by shock velocity and lengths of the chambers.

The pressure ratio across the diaphragm in a shock tube will govern the velocity (Mach number) of the gas flow produced. A wide range of velocities can be made available by this means. The absolute values of pressure in the chambers before burst determine the pressure in the flow zones. The shock tube may be constructed for vacuum operation of the expansion chamber. By controlling both the absolute-pressure section and the pressure ratio, the pressure increase across the shock front can be adjusted to produce a normal one atmosphere value in the flow zones, regardless of the velocity. When pressure ratio and absolute pressure are fixed, the resultant temperature rise also becomes fixed. External preheating or cooling of the chambers is required for temperature control. The temperature of zone T_1 will always be higher and that of zone T_2 lower than the initial ambient temperature by amounts depending on the pressure ratio.

Figure 5 illustrates the uniformity in air particle velocity, temperature, and pressure available in the shock tube. This figure represents these parameters at a particular time after the rupture of the diaphragm. Considering the air particle velocity in respect to the position along the tube axis, one sees on the right hand side of the figure that the velocity is zero. Moving back along the tube axis we find first the shock front and at this point the velocity suddenly rises to its maximum. The shock front is represented by a vertical incline or wall. This maximum velocity is constant to the portion just before the expansion wave, thus giving a region that is uniform in velocity throughout a large portion of the shock tube. This region grows even larger as the shock front progresses forward in the tube and as the expansion wave approaches the rear of the shock tube.

SHOCK TUBE DESCRIPTION

As stated previously the shock tube is similar to the blast gun, except that it is closed on both ends allowing compression of the driver chamber and evacuation of the expansión chamber. The shock tube was mounted vertically so that the test drop could be set in free-fall motion down the expansion chamber where it is met by the shock.

The shock tube, constructed of 2-in. aluminum heavy-wall tubing, has a 2-ft. long compression or driver chamber and a 4-ft. long expansion chamber. The latter contains two $\frac{1}{4}$-in. thick glass-wall photographic sections, each 6 in. in length, which together with cover and spacing plates may be assembled in any of several configurations to permit observation at any point along the length of the expansion chamber. These windows modify the circular cross section of the tube; therefore, wood filler blocks were placed in both chambers to give a fixed area

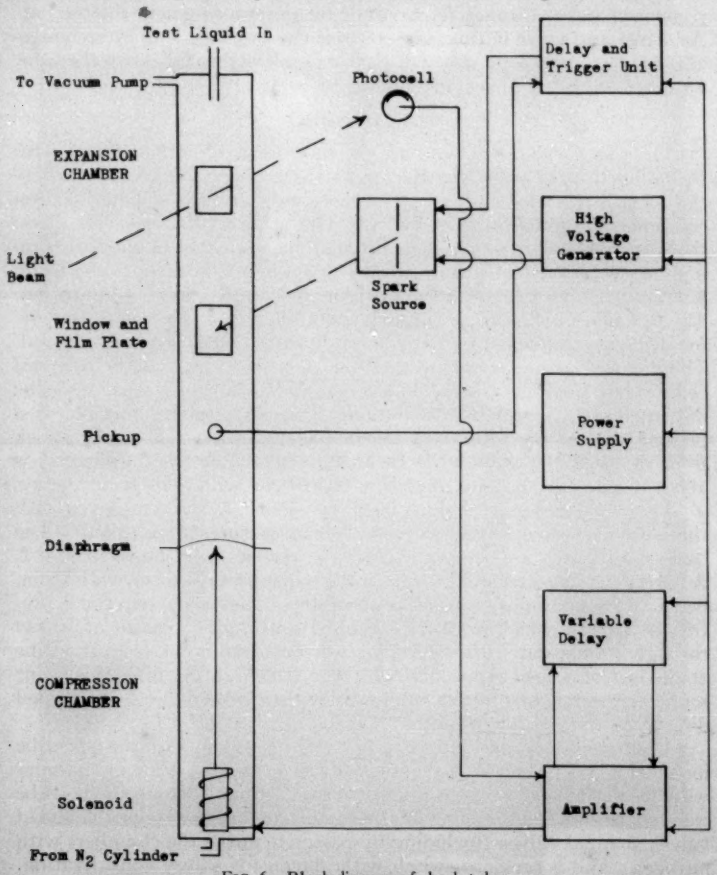


FIG. 6. Block diagram of shock tube.

and shape throughout the tube. This precaution was necessary to assure a constant mass rate of flow along the tube and to prevent formation of extraneous shocks. The lower tube section, the compression chamber, contains the dagger and solenoid to burst the diaphragm.

The two chambers, compression and expansion, are connected by a cammed quick-opening coupling. An expendable diaphragm is placed between the flanges of the coupling for each shot.

Drops are introduced into the center of the partially evacuated expansion chamber through hypodermic tubing entering near the top cap. An orifice and valve in this tube controls the flow rate, and interchangeable needles control the size of drop that is released to fall down the tube. Drops of almost spherical configuration are formed by this method.

INSTRUMENTATION

The shock wave traveling up the tube must be synchronized with the falling drop in order that they meet at the observation section where the event is recorded on film. The shock tube and the synchronization equipment are illustrated in Fig. 6. The chamber has been set up so that a narrow light beam passes through the upper set of windows into a photoelectric cell. A drop passing this window interrupts the beam sending a pulse to a multivibrator-delay circuit after amplification. The function of this delay, variable from 50 μs. to 300 μs., is to provide for adjustment of the drop position in the tube as the shock is released. The delayed pulse discharges a 120 μf. capacitor through the solenoid coil. The dagger pierces the diaphragm and initiates its rupture. The minimum elapsed time for this sequence from beam interruption, at a point 32 in. up the tube from the diaphragm, to arrival of the shock front at the lower window, 18 in. up the tube from the diaphragm, is about 4 μs. under normal operating conditions with zero-preset delay.

The shadowgraph technique used was similar to that employed with the blast gun, except that only single-flash pictures were taken. The basic components are the same as for the double-flash shown in Fig. 2. A barium titanate crystal pickup in the expansion chamber wall 3.5 in. below the first window sensed the shock-front passage to trip the delay-trigger circuits, which in turn discharged a 0.02 μf. capacitor at 10 kv. through a Liebessart gap. This gap was mounted 36 in. from the tube at the end of a light-tight channel. The back lighting from this point source passed through the field windows to a film holder attached directly to the window frame.

Additional accessory equipment needed to operate the shock tube included a d-c. power supply, high voltage generator, nitrogen cylinder with regulator, and vacuum pump. Pressure and vacuum in the tube were controlled from a panel board containing pressure gages, solenoid valves, manual valves (including by-passes to purge the chambers with nitrogen), and a pressure switch with alarms for safety consideration.

DROP BREAKUP BY THE SHOCK-TUBE TECHNIQUE

The shock tube was constructed in order to remove the undesirable effects previously described. To obtain atmospheric pressure in the air-flow zones, the required chamber pressures were 78.8 and 3.94. psi. absolute. The air velocity selected for preliminary tests was 1450 ft./sec. Nitrogen was used as the test medium. The shock-front velocity was about 2290 ft./sec. and the Mach numbers was 1.28 and 2.03 for zones 1 and 2, respectively.

Figure 7 is a series of shadowgraphs showing the breakup of a water

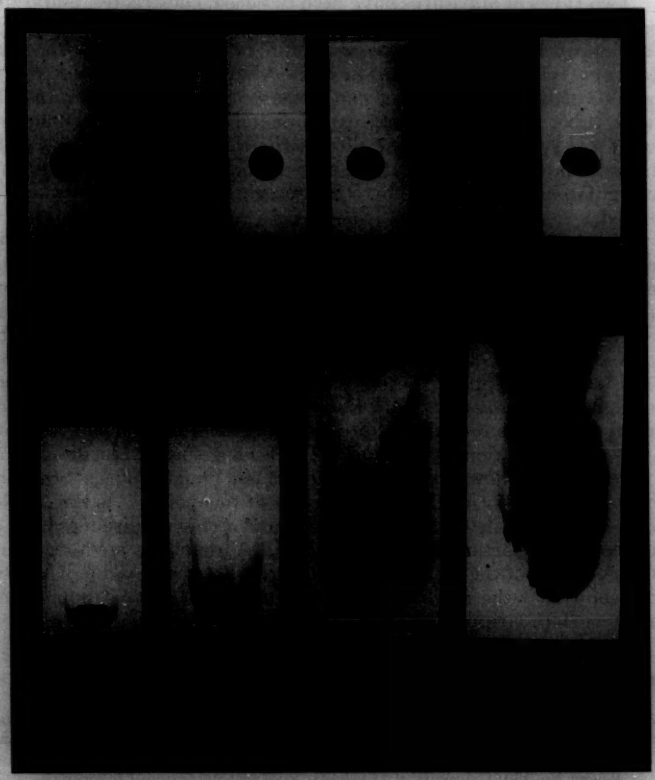


FIG. 7. Shadowgraph series of the breakup of a water drop. Time interval after initial exposure to shock wave given in microseconds. (Reduced ⅓ in reproduction.)

drop 4 mm. in diameter. In these shadowgraphs, about 40 μs. after the drop was struck by the high velocity airstream, the drop had developed surface ripples, indicating that the mechanism of breakup would be by surface stripping. In less than 100 μs., the drop had undergone considerable surface stripping and had formed relatively large quantities of aerosol and vapor, and the particles thus formed had been accelerated ahead of the main particle bulk and were being surface stripped themselves.

DISCUSSION

Two types of apparatus have been designed, constructed, and tested for the study of the mechanism of high velocity breakup of liquids. The first of these, the blast gun, was found to be simple and inexpensive. However, its use was found to be limited by the spurious action due to the jet vortex. On the other hand, the second piece of apparatus, the shock tube, gives the desired experimental control and accuracy required for such studies. The flow in the shock tube was found to be uniform and predictable, and the pressure behind the shock front could be regulated to one atmosphere of pressure or whatever pressure was desired.

Although this report is primarily a description of apparatus, some preliminary observations may be discussed based on the information presented. A gradual transition in the breakup mechanism of drops subjected to steady-velocity airstreams occurs as the initial relative air velocity is increased. The "bursting bag" mode demonstrated by Lane (3) becomes more violent and eventually the mechanism referred to as "surface stripping" becomes predominant. The balance between the high air forces acting on the leading surface of a drop, its acceleration (inertia), and the restraining forces of interfacial tension and viscosity result in deformation of the drop. The leading surface remains convex upstream as the downstream side flattens to a planar shape. Stripping of small droplets from the surface begins at this point. As the droplet cloud grows, the deformations increase and a chaotic disintegration of the remaining particle bulk occurs until the fragments are accelerated to the velocity of the air flow. Shadowgraphs of the breakup of a water drop show that the mechanism of breakup at 1200 ft./sec. is predominantly surface stripping.

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MAXIMUM ANTENNA GAIN OF SHAPED BEAMS *

BY

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SUMMARY

Most of the energy radiated by an antenna lies outside the 3-db. contour of the antenna power pattern. The fraction of the total power which is within the region of concern, not necessarily restricted to the 3-db. points, is termed the concentration factor. The maximum antenna gain of narrow shaped beams, such as the cosecant-squared pattern and its various modifications, can be evaluated within 1 db. if the concentration factor is assumed to be 0.4 for the specific limiting contour which is selected; that is, 60 per cent of the transmitted energy lies outside this region in space. It is unlikely that the measured gain will exceed the answer obtained with this value of the concentration factor by more than 1 db. On the other hand, if the computed gain exceeds the measured value by more than 1 db., it is probable that the design can be improved.

The principal result relates the maximum antenna gain of a shaped beam G_M to the concentration factor $\eta(\Sigma)$ with the equation

$$G_M = \frac{22.2\eta(\Sigma)}{B_H(B_V + 0.715I_2)}$$

where B_H and B_V are the horizontal and vertical beamwidths, respectively. The quantity I_2 is dependent on the beam shape in the vertical plane, as defined in Eq. 16.

For a transmitter located at the origin of a coordinate system, the power emitted into a region of space bordered by a cone having its vertex at the origin is

$$P(\Sigma) = \int_{\Sigma} S_r \mathbf{a}_r \cdot \mathbf{d\sigma} \tag{1}$$

when the distance from the source is sufficiently great for the Poynting vector to be essentially radial in direction. In this equation

$P(\Sigma)$ = power emitted into the conical region bordered by the surface Σ.
S_r = radial component of the Poynting vector.
$\mathbf{a}_r$ = unit vector in the radial direction.
$\mathbf{d\sigma}$ = vector element of area in the surface Σ.

* This paper is adapted from Technical Report AFTR-6505, Wright Air Development Center, Wright-Patterson Air Force Base, Ohio, March, 1951.

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For free-space propagation and negligible atmospheric attenuation,

$$S_r = \frac{Pg(\theta, \phi)}{4\pi r^2}, \tag{2}$$

where P is the total radiated power, r is the radial distance from the antenna, and $g(\theta, \phi)$ is the power gain of the antenna expressed as a function of the spherical coordinate angles.

Substitution of Eq. 2 puts Eq. 1 into the form

$$P(\Sigma) = \frac{P}{4\pi} \int_\Sigma g(\theta, \phi) \frac{\mathbf{a}_r \cdot \mathbf{d\sigma}}{r^2}. \tag{3}$$

Now the concentration factor, $\eta(\Sigma)$, is defined [2] as the fraction of the radiated power that passes through the surface Σ, so that

$$\eta(\Sigma) = \frac{P(\Sigma)}{P} = \frac{1}{4\pi} \int_\Sigma g(\theta, \phi) \frac{\mathbf{a}_r \cdot \mathbf{d\sigma}}{r^2}. \tag{4}$$

For an integration that involves spherical coordinates

$$\mathbf{a}_r \cdot \mathbf{d\sigma} = r^2 \sin\theta \, d\theta d\phi = -r^2 \cos\beta \, d\beta d\phi$$

where ϕ is the azimuth angle, θ is colatitude (the polar angle), and $\beta = \pi/2 - \theta$ is latitude. Then the concentration factor may be expressed as

$$\eta(\Sigma) = \frac{1}{4\pi} \int_\Sigma g(\theta, \phi) \sin\theta \, d\theta d\phi = -\frac{1}{4\pi} \int_\Sigma g(\beta, \phi) \cos\beta \, d\beta d\phi. \tag{5}$$

CONCENTRATION FACTOR FOR PENCIL BEAMS

For a pencil beam having equal horizontal and vertical beamwidths, the variation of the antenna power gain in the region of the peak of the pattern is of a Gaussian nature,[3] being approximated by

$$g = G_M e^{-0.69315(\theta/\theta_b)^2} \qquad \text{for} \qquad \theta \leqq \theta_b, \tag{6}$$

where G_M is the maximum power gain, θ is the polar angle, and θ_b is

[2] The term "concentration factor" was introduced by B. Dossier, according to a footnote on p. 38 of J. J. Myers, "Antenna Image Quality Criteria," *Control Systems Laboratory Report R-108*, University of Illinois, Urbana, Ill., Dec. 1958. It appears preferable to the term "directive efficiency" employed in the original version of this paper, because of other meanings of "directive" in the antenna field.

[3] A better approximating function for the main lobe and near side lobes has been suggested in N. D. Diamantides, "Finding Radar Pattern Equations with a Computer," *Electronic Industries*, Vol. 19, pp. 89–91 (1960). The form employed by Diamantides is

$$g = G_M[\exp[-a(\theta/\theta_b)^2] + m \exp(-b\theta/\theta_b) \sin^2(c\theta/\theta_b)].$$

For the very limited portion of the pattern lying within the half-power contour, however, the simple Gaussian is a satisfactory approximation for the purposes of this paper.

the half-power angle. When the integration is carried out over the surface contained within the half-power contour, Eq. 5 becomes

$$\eta(\Sigma_1) = \frac{G_M}{4\pi} \int_0^{\theta_b} \int_0^{2\pi} e^{-0.69315(\theta/\theta_b)^2} \sin\theta \, d\theta d\phi$$

$$= 0.04509 G_M B_\theta^2, \tag{7}$$

where $B_\theta = 2\theta_b$ is the beamwidth between half-power points, and Σ_1 is a surface bounded by the half-power contour. (We assume θ_b to be small in evaluating the integral.)

For parabolic reflectors a more useful relation is obtained if we let

$$G_M = \alpha_g \pi^2 \left(\frac{L_a}{\lambda}\right)^2,$$

where

L_a = diameter of the parabolic reflector
λ = wavelength
α_g = empirical constant

and

$$B_\theta = \alpha_\theta \frac{\lambda}{L_a},$$

where α_θ is an empirical constant for a given type of antenna[4]; then Eq. 7 becomes

$$\eta(\Sigma_1) = 0.04509 \pi^2 \alpha_g \alpha_\theta^2 = 0.4392 \alpha_g \alpha_\theta^2. \tag{8}$$

For example, for a parabolic reflector with a dipole feed,[5] $\alpha_g = 0.65$ and $\alpha_\theta = 1.2$, so that the concentration factor is

$$\eta(\Sigma_1) = 0.415. \tag{9}$$

This result indicates that most of the transmitted power (58.5 per cent) is radiated beyond the half-power points for the antenna pattern discussed.

We shall assume that the results are not changed significantly when the beamwidths are not equal, so that Eq. 7 may be written in a more general form as

$$\eta(\Sigma_1) = 0.04509 G_M B_V B_H \tag{10}$$

[4] In the equation for the maximum gain, α_g is the *gain factor* defined in S. Silver, "Microwave Antenna Theory and Design," New York, McGraw-Hill Book Co., Inc., 1949, p. 178. In the equation for the beamwidth, α_θ corresponds to the constants in the columns for beamwidth in Tables 6.1 (p. 187) and 6.2 (p. 195) of this reference.

[5] L. C. Van Atta and C. V. Robinson, "Information on Standard Radiation Laboratory Reflectors," MIT Radiation Laboratory Report 54–11/4/42.

where the subscripts V and H identify the vertical and horizontal beamwidths, respectively. Then Eq. 8 becomes

$$\eta(\Sigma_1) = 0.4392\alpha_p\alpha_V\alpha_H, \tag{11}$$

where α_V and α_H apply to the beamwidth constants in the vertical and horizontal.

MAXIMUM GAIN OF SHAPED BEAMS

If a shaped beam concentrates its energy in space as efficiently as a paraboloid, then it has a concentration factor in the neighborhood of 0.4. A somewhat larger factor might be expected because the surface for a shaped beam covers a larger region in space (in one dimension) than the surface for the 3-db. contour of a pencil beam. On the other hand, the added complexity of controlling the energy distribution for the shaped beam results in additional scattered radiation, so that no net improvement is realized.

The vertical pattern of many shaped beams can be represented by a Gaussian function about the peak of the beam, and a controlled angular variation at steeper depression angles. The variation in a vertical plane may then be represented by

$$g(\beta) = \begin{cases} g_1(\beta) = G_M e^{-0.69315(\beta-\beta_1)^2/\theta_b^2}, & \beta_1 - \theta_b < \beta < \beta_1 + \theta_b \\ \dfrac{G_M}{2}\, g_2(\beta), & \beta_1 + \theta_b < \beta < \beta_M \end{cases} \tag{12}$$

where β is the depression angle; β_1 is the angle of the peak of the beam at which the gain attains its maximum value G_M; θ_b is the angle measured from β_1 at which the gain is 3 db. below its maximum; and β_M is the largest depression angle over which the pattern is controlled. The gain function of the shaped region $g_2(\beta)$ is normalized to unity at the angle $\beta = \beta_1 + \theta_b$.

For the two-dimensional variation of the gain, the Gaussian region is represented by $g_1(\beta, \phi)$ and the shaped section is $g_2(\beta, \phi)$. With these functions Eq. 5 becomes

$$\eta(\Sigma) = -\frac{1}{4\pi}\int_{\Sigma_1} g_1(\beta, \phi)\cos\beta\, d\beta d\phi - \frac{1}{4\pi}\int_{\Sigma_2} g_2(\beta, \phi)\cos\beta\, d\beta d\phi \tag{13}$$

where Σ_2 is a surface extending over the region

$$\beta_1 + \theta_b < \beta < \beta_M$$

$$-\frac{\phi_b}{\cos\beta} < \phi < \frac{\phi_b}{\cos\beta}.$$

The first term in Eq. 13 has been evaluated, being given by Eq. 10. To handle the second integral, we shall assume that the conical surface β = constant has a Gaussian intercept with $g_2(\beta, \phi)$; then the complete expression for $g_2(\beta, \phi)$ is

$$g_2(\beta, \phi) = \frac{G_M}{2} g_2(\beta) e^{-0.69315(\phi \cos \beta/\phi_b)^2} \tag{14}$$

so that the second term in Eq. 13, represented by I_1, is

$$I_1 = -\frac{G_M}{8\pi} \int_{\Sigma_2} g_2 \cos \beta \, d\beta d\phi$$

$$= -\frac{G_M}{8\pi} \int_{\beta_M}^{\beta_1+\theta_b} \int_{-\phi_b/\cos\beta}^{\phi_b/\cos\beta} e^{-0.69315(\phi \cos \beta/\phi_b)^2} g_2(\beta) \cos \beta \, d\phi d\beta.$$

The substitution

$$\frac{t}{\sqrt{2}} = \frac{\sqrt{.69315}\phi \cos \beta}{\phi_b}$$

reduces the integral to

$$I_1 = \frac{\phi_b G_M}{4\sqrt{2\pi}(1.1774)} \frac{1}{\sqrt{2\pi}} \int_{-1.1774}^{1.1774} e^{-t^2/2} dt \int_{\beta_1+\theta_b}^{\beta_M} g_2(\beta) d\beta$$

$$= \frac{B_H G_M P(1.1774)}{8\sqrt{2\pi}(1.1774)} I_2 = 0.03223 B_H G_M I_2 \tag{15}$$

where $P(x)$ is the normal probability integral, and

$$I_2 = \int_{\beta_1+\theta_b}^{\beta_M} g_2(\beta) d\beta. \tag{16}$$

Substitution of Eqs. 10 and 15 into 13 leads to the expression for the maximum gain of the shaped beam pattern:

$$G_M = \frac{22.2\eta(\Sigma)}{B_H(B_V + 0.715 I_2)}. \tag{17}$$

Example 1—Cosecant-Squared Beam

For a cosecant-squared beam,

$$g_2(\beta) = \frac{\csc^2 \beta}{\csc^2 (\beta_1 + \theta_b)} \tag{18}$$

so that Eq. 16 becomes

$$I_2 = \sin^2(\beta_1 + \theta_b) \int_{\beta_1+\theta_b}^{\beta_M} \frac{d\beta}{\sin^2\beta}$$

$$= \sin^2(\beta_1 + \theta_b)[-\cot\beta_M + \cot(\beta_1 + \theta_b)]$$

$$\equiv \frac{\sin(\beta_1 + \theta_b)}{\sin\beta_M} \sin(\beta_M - \beta_1 - \theta_b). \tag{19}$$

Then Eq. 17 gives the maximum gain of a cosecant-squared pattern as

$$G_M = \frac{22.2\eta(\Sigma)}{B_H\left[B_V + \dfrac{0.715\sin(\beta_1 + \theta_b)\sin(\beta_M - \beta_1 - \theta_b)}{\sin\beta_M}\right]}. \tag{20}$$

Example 2—Modified Cosecant-Squared Pattern

One form of a modified cosecant-squared beam has

$$g_2(\beta) = \frac{\csc^2\beta\sqrt{\cos\beta}}{\csc^2(\theta_b + \beta_1)\sqrt{\cos(\theta_b + \beta_1)}}. \tag{21}$$

Then Eq. 16 may be written as

$$I_2 \csc^2(\theta_b + \beta_1)\sqrt{\cos(\theta_b + \beta_1)} = \int_{\theta_b+\beta_1}^{\beta_M} \csc^2\beta\sqrt{\cos\beta}\, d\beta$$

$$\equiv \frac{1}{2}\int_{\theta_b+\beta_1}^{\beta_M} \frac{2\sin\beta\cos\beta\, d\beta}{\sin^2\beta\cos^{1/2}\beta}$$

$$= \frac{1}{2}\int_{t'}^{t''} t^{-3/4}(1-t)^{-1/2}dt$$

where

$$t = \sin^2\beta$$

$$t' = \sin^2(\theta_b + \beta_1)$$

$$t'' = \sin^2\beta_M.$$

The integral may be expressed as the incomplete Beta Function, whence

$$I_2 = \frac{\sin^2(\theta_b + \beta_1)}{2\sqrt{\cos(\theta_b + \beta_1)}}[B_{t''}(\tfrac{1}{4}, \tfrac{1}{2}) - B_{t'}(\tfrac{1}{4}, \tfrac{1}{2})] \tag{22}$$

which may be substituted directly into Eq. 17 to determine the maximum gain.

Example 3—Modified Cosecant-Squared Pattern

Another common modification of the cosecant-squared pattern is expressed by

$$g(\beta) = \frac{\csc^2 \beta \cos \beta}{\csc^2 (\theta_b + \beta_1) \cos (\theta_b + \beta_1)}. \tag{23}$$

For this function

$$I_2 \csc^2 (\theta_b + \beta_1) \cos (\theta_b + \beta_1) = \int_{\theta_b+\beta_1}^{\beta_M} \frac{\cos \beta}{\sin^2 \beta} d\beta$$

$$= \frac{1}{\sin (\theta_b + \beta_1)} - \frac{1}{\sin \beta_M}$$

whence

$$I_2 = \tan (\theta_b + \beta_1) \left[1 - \frac{\sin (\theta_b + \beta_1)}{\sin \beta_M} \right] \tag{24}$$

and the maximum gain is

$$G_M = \frac{22.2\eta(\Sigma)}{B_H \left\{ B_V + 0.715 \tan (\theta_b + \beta_1) \left[1 - \frac{\sin (\theta_b + \beta_1)}{\sin \beta_M} \right] \right\}}. \tag{25}$$

Illustrative Example: For $B_H = 2.5°$, $B_V = 6°$, $\beta_1 + \theta_b = 6°$ and $\beta_M = 66°$, the gain with $\eta(\Sigma) = 0.4$ is

$$G_M = 1190.$$

The measured gain of an antenna with this pattern[6] was 1380, which is only 0.7 db. greater.

CONCLUSIONS

In general we may expect well-designed antennas having similar patterns to have approximately the same concentration factors, provided they include nearly equal solid angles. Consequently, the equations presented should represent the maximum gain of shaped beam patterns within, say, 1 db. when the concentration factor is set equal to 0.40. In effect, this tolerance of 1 db. corresponds to

$$0.32 \leq \eta(\Sigma) \leq 0.50$$

[6] A. S. Dunbar, "Calculation of Doubly Curved Reflectors for Shaped Beams," *Proc. IRE*, Vol. 36, pp. 1289–1296 (1948), see p. 1294.

which is a physically reasonable range of values for narrow-beam patterns.

For the measured gain of a narrow-beam antenna to exceed the computed value based on an assumed concentration factor of 0.4 by more than 1 db., it is necessary that less than 50 per cent of the energy be beyond the surface defined in this paper. This condition may be realizable if the illumination of the source is closely controlled with reversed phase toward the edges, but appears very unlikely for normal design. If, however, the reverse condition should be found, with the computed gain more than 1 db. in excess of the measured value, then at least 68 per cent of the emitted energy is beyond the surface which has been defined. This condition implies that better design could be employed to control the spatial energy distribution.

THE ANALYSIS OF A VERTICAL AND A HORIZONTAL CANTILEVER UNDER A UNIFORMLY DISTRIBUTED LOAD

BY

R. FRISCH-FAY[1]

SUMMARY

The buckling of a bar under its own weight and the large deflections of a cantilever under a uniformly distributed load and analyzed from a unified approach. Border cases for which the approximate theory is applicable are discussed.

INTRODUCTION

The buckling of a bar under its own weight was first investigated by Euler but he did not succeed in giving a satisfactory solution. The problem was first solved by A. G. Greenhill in 1881. The solution was based on the approximate theory, *viz.* $1/r = d^2y/dx^2$.

The nonlinear bending of a cantilever under its own weight was first investigated by Hummel and Morton (1).[2] A series solution was proposed which, however, failed to converge fast enough for values larger than 45° of the tangent at the free end. Bickley (2) used numerical methods to solve the basic differential equation for the deflection of the end of the cantilever and tabulated the results at intervals of 10° for the end tangent up to 70°. Rohde (3) used an infinite series to establish a relationship between the arc length and the tangent. Similar problems of a more complex nature were discussed by K. T. Sundara Raja Iyengar (4) and by K. T. Sundara Raja Iyengar and S. K.Lakshmana Rao (5).

The basic difficulty research has encountered in this type of problems was the question of finding an approximate solution for a nonlinear second order differential equation with variable coefficient.

BUCKLING OF A BAR UNDER ITS OWN WEIGHT (APPROXIMATE THEORY)

According to the approximate theory the buckling of a vertical bar under its own weight, fixed at the bottom, free at the top (Fig. 1), is solved by a linear, second order differential equation with a variable coefficient (A. G. Greenhill, 1881),

$$d^2u/dz^2 + k^2zu = 0 \tag{1}$$

where

$$u(z) = dy/dx = \varphi$$
$$z = L - x$$
$$k = \sqrt{(w/EI)}.$$

[1] School of Civil Engineering, University of New South Wales, Sydney, Australia.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

It should be noted that a very flat shape is being assumed for the buckled bar, hence $s \approx z \equiv L - x$. Later in the discussion this provision will no longer be necessary.

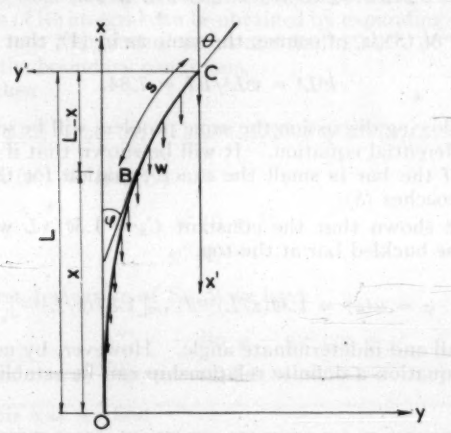

FIG. 1.

The integral of Eq. 1 is

$$u(z) = z^{1/2}[C_1 J_{1/3}(2kz^{3/2}/3) + C_2 J_{-1/3}(2kz^{3/2}/3)] \tag{2}$$

where $J_{1/3}$ is Bessel's function of order 1/3, first kind. We note that the tangent at the fixed end is zero, and that the curvature at the top is also zero. By applying these boundary conditions we find that

$$C_1 = 0$$

and

$$C_2 L^{1/2} J_{-1/3}(2kL^{3/2}/3) = 0. \tag{3}$$

The first zero of $J_{-1/3}$ is 1.87, hence

$$(wL)_{cr} = 7.84EI/L^2. \tag{4}$$

Expanding the Bessel function in Eq. 3 into a series we have

$$J_{-1/3}(2kL^{3/2}/3) = \frac{(kL^{3/2}/3)^{-1/3}}{\Gamma(2/3)} \times \left[1 - \frac{3(kL^{3/2}/3)^2}{2} + \frac{3^2(kL^{3/2}/3)^4}{2!2\cdot 5} - \frac{3^3(kL^{3/2}/3)^6}{3!2\cdot 5\cdot 8} + \right]\cdots = 0.$$

The series in the bracket can be reduced to

$$1 - \frac{k^2L^3}{2\cdot 3} + \frac{k^4L^6}{2\cdot 3\cdot 5\cdot 6} - \frac{k^6L^9}{2\cdot 3\cdot 5\cdot 6\cdot 8\cdot 9} + \cdots = 0. \tag{5}$$

The solution of (5) is, of course, the same as in (4), that is

$$k^2L^3 = wL^3/EI = 7.84.$$

In the following discussion the same problem will be solved by using the exact differential equation. It will be shown that if the deflection of the top of the bar is small the exact equation for the solution of wL^3/EI approaches (5).

It can be shown that the constant $C_2 = 1.3\theta/\sqrt{L}$ where θ is the tangent of the buckled bar at the top.

Hence,

$$\varphi = u(z) = 1.3\theta(z/L)^{1/2}J_{-1/3}[1.87\,(z/L)^{3/2}]. \tag{6}$$

θ is a small and indeterminate angle. However, by using the exact differential equation a definite relationship can be established between θ and wL.

BUCKLING OF BAR UNDER ITS OWN WEIGHT (EXACT THEORY)

In order to solve the same problem according to the exact theory we first transfer the coordinate system into the $x'y'$ position.

The exact differential equation governing the slope of the buckled bar is

$$EI\,d\varphi/ds = -M. \tag{7}$$

Differentiating with respect to ds, and rearranging we have

$$d^2\varphi/ds^2 = -ws\sin\varphi/EI = -q\sin\varphi/c^2 \tag{8}$$

where $c = (EI/w)^{1/3}$ and $q = s/c$.

Noting that

$$d^2\varphi/ds^2 = c^{-2}d^2\varphi/dq^2$$

we reduce (8) to

$$d^2\varphi/dq^2 = -q\sin\varphi. \tag{9}$$

The boundary conditions will state that at the top of the bar the curvature is zero and the tangent to the curve is θ.

We have then

$$\frac{1}{c}\,d\varphi/dq\Big|_{q=0} = 0 \tag{a}$$

and

$$\varphi|_{q=0} = \theta. \qquad (b)$$

Equation 9 does not seem to admit of a closed form solution. An approximation of its integral can be obtained by expanding the unknown function $\varphi(q)$ into Maclaurins' series and find φ', φ'', φ''', etc. in conformity with the boundary conditions.

We have then

$$\varphi(q) = \varphi(0) + q\varphi'(0) + q^2\varphi''(0)/2! + q^3\varphi'''(0)/3! + \cdots. \qquad (10)$$

We have from (a), (b) and (9)

$$\varphi'(0) = 0$$
$$\varphi''(0) = 0.$$

Taking the first derivative of Eq. 9 we have

$$\varphi''' = -\sin\varphi - q\varphi'\cos\varphi \qquad (c)$$
$$\varphi'''(0) = -\sin\theta.$$

Continuing this way we find

$$\left.\begin{aligned} \varphi^{(4)}(0) &= 0 \\ \varphi^{(5)}(0) &= 0 \\ \varphi^{(6)}(0) &= 2\sin\theta \\ \varphi^{(7)}(0) &= 0 \\ \varphi^{(8)}(0) &= 0 \\ \varphi^{(9)}(0) &= 70\sin^3\theta - 14\cos\theta\sin 2\theta. \end{aligned}\right\} \qquad (d)$$

Substituting (a), (b), (c) and (d) into (10) and letting $q^3 = m$, we get the approximate solution

$$\varphi(q) = \theta - m\sin\theta/6 + m^2\sin 2\theta/360 + m^3\sin^3\theta/5184 - m^3\cos\theta\sin 2\theta/25920. \qquad (11)$$

According to the elementary theory, buckling occurs at small values of θ. In this case $\theta \approx \sin\theta$ and $\cos\theta \approx 1$. Substituting these values into Eq. 11, dividing the equation by θ and noting that

$$\varphi|_{q=L/c} = 0$$

we have

$$1 - m_L/2\cdot 3 + m_L^2/2\cdot 3\cdot 5\cdot 6 - m_L^3/2\cdot 3\cdot 5\cdot 6\cdot 8\cdot 9 + \cdots = 0 \qquad (12)$$

where

$$m_L = m|_{s=L} = L^3/c^3 = wL^3/EI = k^2L^3.$$

The series in Eq. 12 is identical with the one in Eq. 5. Solving Eq. 12 for its lowest root, we have

$$m_L = 7.81.$$

Adding one more term

$$m_L{}^4/2\cdot3\cdot5\cdot6\cdot8\cdot9\cdot11\cdot12$$

to Eq. 12 we get the correct value of the root to two decimals

$$m_L = 7.84$$

and $wL = 7.84EI/L^2$ which is the critical load according to the elementary theory.

By using the exact differential equation (9) for the deflection of the bar, the notion of $(wL)_{cr}$ becomes meaningless in the ordinary sense. The bar will not collapse if wL reaches the value of $(wL)_{cr}$, nor will the bar remain in an indeterminately (though slightly) bent shape. When wL increases beyond $(wL)_{cr}$, Eq. 11 will give a definite solution for θ.

At the fixed end

$$\varphi|_{q=L/c} = 0$$

hence

$$\theta - m_L \sin\theta/6 + m_L{}^2 \sin 2\theta/360 + m_L{}^3 \sin^3\theta/5184 - m_L{}^3 \cos\theta \sin 2\theta/25920 = 0 \quad (13)$$

is the transcendental equation to be solved for θ.

If $wL/(wL)_{cr} = 1 + \Delta$, where Δ is small compared with unity,

$$\theta \approx 2.1\sqrt{\Delta}. \quad (14)$$

This approximation has been obtained by neglecting the term

$$m_L{}^3 \sin^3\theta/5184$$

in Eq. 13, by adding a term containing $m_L{}^4$, and by expanding $\sin\theta$ into $\theta - \theta^3/6$. Accordingly, if $\Delta = 0.001$, then $\theta = 0.067$, and if $\Delta = 0.01$, then $\theta = 0.21$.

Once θ is solved from Eq. 13, $\varphi(q)$ can be calculated for any value

of q, that is, of s. The coordinates x' and y' can be solved from

$$x' = \int_0^s \cos \varphi \, ds = c \int_0^q \cos \varphi \, dq \tag{15}$$

$$y' = \int_0^s \sin \varphi \, ds = c \int_0^q \sin \varphi \, dq. \tag{16}$$

Since dq has no formal solution in terms of φ, numerical methods have to be employed to solve Eqs. 15 and 16.

THE HORIZONTAL CANTILEVER

Considering now a horizontal cantilever under a distributed load (Fig. 2) we note that

$$\theta - \tfrac{1}{2}\pi = \alpha.$$

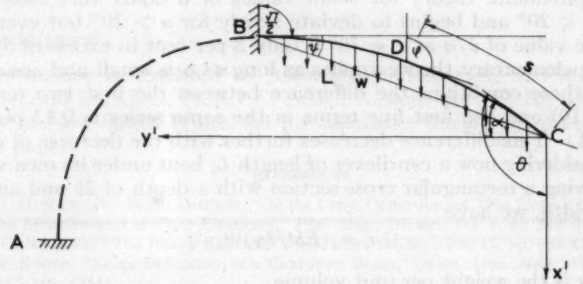

FIG. 2.

Hence, from Eq. 11 we have

$$\tfrac{1}{2}\pi = (\alpha+\tfrac{1}{2}\pi) - n_L \sin(\alpha+\tfrac{1}{2}\pi)/6 + n_L^2 \sin(2\alpha+\pi)/360 + n_L^3 \sin^3(\alpha+\tfrac{1}{2}\pi)/5184 - n_L^3 \cos(\alpha+\tfrac{1}{2}\pi)\sin(2\alpha+\pi)/25920 \tag{17}$$

where $n_L = L^3/c^3$.

After some reductions we have

$$\alpha - n_L \cos\alpha/6 - n_L^2 \sin 2\alpha/360 + n_L^3 \cos^3\alpha/5184 - n_L^3 \sin\alpha \sin 2\alpha/25920 = 0 \tag{18}$$

for the solution of α.

The relationship between $q = s/c$ and φ is expressed by

$$\varphi(q) - \tfrac{1}{2}\pi = \alpha - n\cos\alpha/6 - n^2\sin 2\alpha/360 + n^3\cos^3\alpha/5184 - n^3\sin\alpha\sin 2\alpha/25920 \quad (19)$$

where $n = s^3/c^3$.

The coordinates of any point of the cantilever can be found from

$$x' = c\int_0^q \cos\varphi\, dq \quad (20)$$

$$y' = c\int_0^q \sin\varphi\, dq. \quad (21)$$

For small deflections we find that

$$\psi = 3\alpha/4. \quad (22)$$

Investigations by Bickley (2) have shown that Eq. 22 derived from the approximate theory for small values of α holds very closely for $0° < \alpha < 20°$ and begins to deviate slowly for $\alpha > 20°$ but even then the true value of ψ/α at $\alpha = 70°$ is only 8 per cent in excess of 3/4.

The elementary theory applies as long as α is small and $\cos\alpha \approx 1$. Under these conditions the difference between the first two terms in series (19) and the first five terms in the same series is 0.12 per cent if $q = 1$. This difference decreases further with the decrease of q.

Considering now a cantilever of length L, bent under its own weight and having a rectangular cross section with a depth of $2b$ and an arbitrary width, we have

$$c = (Eb^2/3\gamma)^{1/3}$$

where γ is the weight per unit volume.

Hence, the condition of $q > 1$ is

$$\frac{L}{(Eb^2/3\gamma)^{1/3}} > 1. \quad (23)$$

Let $b = \beta L$ and $E/\gamma = \eta L$. It follows from (23) that

$$\beta > \sqrt{(3/\eta)}. \quad (e)$$

On the other hand the bending moment at the fixed end

$$M = L^2w/2 = fI/b$$

where I = moment of inertia and f = extreme fibre stress. It follows that

$$f = 1.5L\gamma/\beta. \quad (f)$$

However, in order to remain in the elastic range

$$f < f_{yield},$$

therefore, from (f) we have

$$f_y > 1.5L\gamma/\beta$$
$$\beta > 1.5L\gamma/f_y. \qquad (g)$$

The bar must fulfill the conditions expressed by (e) and (g) in order to produce deflections sufficiently small to warrant the use of the simplified theory.

If

$$(3/\eta)^{1/2} > \beta > 1.5L\gamma/f_y$$

the stresses at the fixed end are not necessarily above yield point because the resulting large deflections will give a nonlinear relationship between L^3 and M.

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APPLICATION OF GRAPH THEORY TO THE ANALYSIS OF ACTIVE AND MUTUALLY COUPLED NETWORKS *

BY

WAN HEE KIM [1]

SUMMARY

This paper presents a unified graphical approach to the analysis of an electrical network which contains linear multi-terminal devices such as vacuum tubes, transistors, transformers, gyrators, circulators, etc. The concept of the mathematical equivalent circuit of an active device or a passive but mutually coupled device which was introduced by Percival, is related to the indefinite admittance matrix of a multipole defined by Zadeh and Shekel.

Various graphical formulas for the evaluation of network functions of a multipole are shown, including the node-condensation techniques. A number of examples are presented.

INTRODUCTION

The method of topological analysis (application of the theory of linear graphs) of a network comprising linear, passive and bilateral two-terminal devices such as resistors, inductors and capacitors, has been proved very useful (1, 2).[2] It is particularly suitable for digital computers (3). However, the method is not directly applicable to a network containing linear active devices, such as vacuum tubes and transistors, or mutually coupled passive devices, such as transformers and gyrators. The difficulty is the graphical representation of mutually dependent node-pairs due to a dependent source or the mutual coupling effect imbedded in a device (4). In order to characterize a dependent node-pair in a tube and transformer by graphical representation, Percival (5, 6) recently introduced artificial two-terminal elements, so-called "current and voltage elements," and hence made it possible for the theory of linear graphs to be extended to the analysis of active and mutually coupled networks (7, 8).

This paper presents first an expository review and extension of the concept and use of "current and voltage elements" in graphical characterization of dependent node-pairs in a network. Then, through investigation of the topological aspects of the techniques of multipole analysis proposed by Zadeh (9, 10) and Shekel (11, 12), a unified approach to the analysis of an electrical network containing multiterminal active and passive devices is pursued.

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

1. GRAPHICAL CHARACTERIZATION OF MUTUALLY DEPENDENT NODE-PAIRS

Let us consider node-pairs (pq) and (mn) in the linear network N shown in Fig. 1. If the current $i_{pq}(t)$ flowing from node p to node q depends not only on the voltage-difference between nodes p and q,

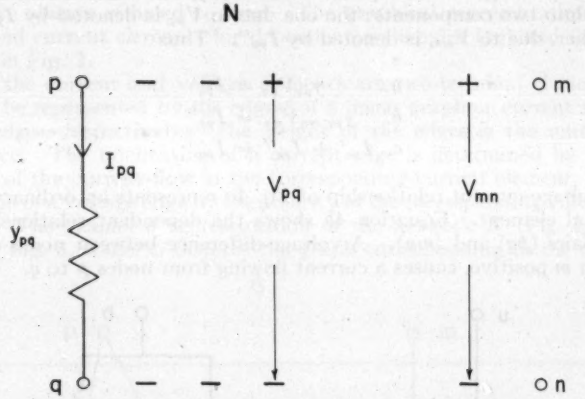


FIG. 1. A linear network with mutually dependent node-pairs.

$v_{pq}(t)$, but also on the voltage difference between nodes m and n, $v_{mn}(t)$, then, since the network is linear, one may write

$$y_{pq}V_{pq} + y_{pq,mn}V_{mn} = I_{pq} \tag{1}$$

where the y's are proportional factors and $I = \mathfrak{L}i(t)$, $V = \mathfrak{L}v(t)$.

In Eq. 1, since y_{pq} relates the voltage and current for the same node pair, it is called the "self-admittance" or simply "admittance" of the element (or elements) connected between nodes p and q. On the other hand, $y_{pq,mn}$ relates the voltage and current for different node-pairs, and hence is called the "mutual admittance."

If the network N consists entirely of passive devices, then Eq. 1 is divided into two isolated parts, that is,

$$y_{pq}V_{pq} = I_{pq}$$

or (2)

$$y_{pq,mn}V_{mn} = I_{pq}.$$

It is of course clear that if N contains only the ordinary two-terminal devices (R, L and C elements), then $y_{pq} = y_{qp}$ for all node-pairs in the network, which is said to be "passive and reciprocal." However, if

transformers or gyrators are contained in N, then for some node-pairs in the network $y_{pq,mn} \neq y_{mn,pq}$, and N is called "passive and non-reciprocal." Therefore, Eq. 1 holds for some node-pairs in N if it contains a linear active device or devices.

In Eq. 1, I_{pq}, which depends on both V_{pq} and V_{mn}, can be decomposed into two components: the one due to V_{pq} is denoted by I_{pq}' and the other, due to V_{mn}, is denoted by I_{pq}''. Thus

$$\begin{aligned} &a. \quad y_{pq}V_{pq} = I_{pq}' \\ &b. \quad y_{pq,mn}V_{mn} = I_{pq}'' \\ &c. \quad I_{pq}' + I_{pq}'' = I_{pq}. \end{aligned} \tag{4}$$

The voltage-current relationship of Eq. 4*a* represents an ordinary two-terminal element. Equation 4*b* shows the dependent relationship of node-pairs (pq) and (mn). A voltage-difference between nodes m and n, with m positive, causes a current flowing from nodes p to q.

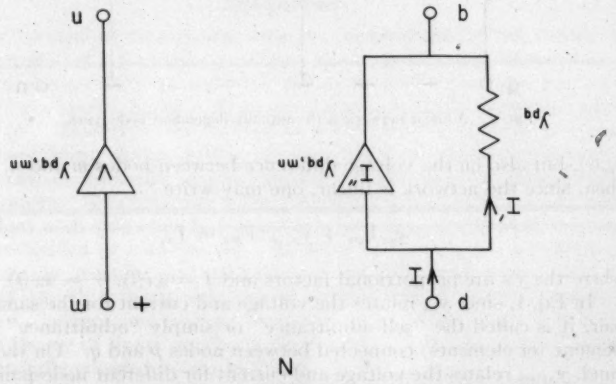


FIG. 2. Use of voltage and current elements to represent dependent node-pairs.

In order to apply the concept of graphs, it is therefore desirable to introduce artificial two-terminal elements into node-pairs (mn) and (pq) to characterize their dependent relationship. The element inserted between nodes m and n, called a "voltage element," is to sense a voltage-difference in the node-pair. The element introduced in node-pair (pq), called a "current element," produces a current of magnitude I'' flowing from node p to node q due to the voltage-difference V in node-pair (mn). The voltage and current elements are thus related by mutual admittance $y_{pq,mn}$. Note that voltage and current elements always occur in a pair

and between different node-pairs. If they occur in the same node-pair, that is, $p = m$ and $q = n$, the mutual admittance $y_{pq,mn}$ is reduced to the self-admittance y_{pq}. Note also that the voltage and current elements introduced cannot be reduced to zero in the evaluation of a network function although all independent voltage and current generators are always reduced to zero. The graphical representation of voltage and current elements for dependent node-pairs (pq) and (mn) is shown in Fig. 2.

Since the current and voltage elements are two-terminal elements they can be represented by the edges[3] of a linear graph as current and voltage edges, respectively. The weight of the edges is the mutual admittance. The orientation of a current edge is determined by the direction of the current-flow in the corresponding current element, the orientation of a voltage edge by the polarity of the corresponding voltage element. The graphical representation of the network N (Fig. 2) is shown in Fig. 3, where G denotes the graph corresponding to the net-

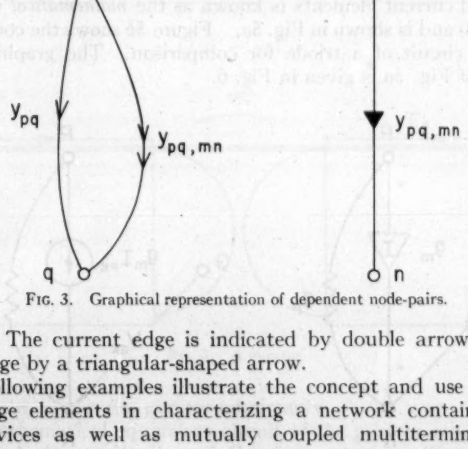


FIG. 3. Graphical representation of dependent node-pairs.

work N. The current edge is indicated by double arrows, and the voltage edge by a triangular-shaped arrow.

The following examples illustrate the concept and use of current and voltage elements in characterizing a network containing linear active devices as well as mutually coupled multiterminal passive elements.

Example 1. A triode of Fig. 4 is characterized as a linear active device by

$$I_P = g_m V_{GK} + g_p V_{PK} \tag{4}$$

[3] The term "branch" may be substituted occasionally for "edge," but the term "element" will only be used in the sense of "element of a network" or "element of a matrix."

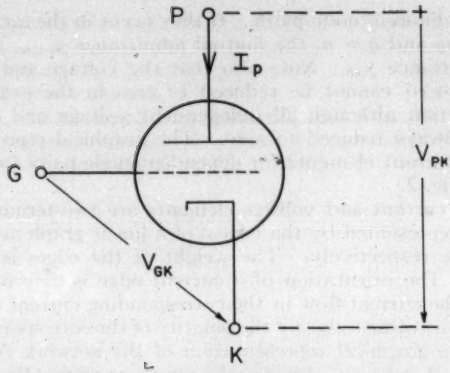
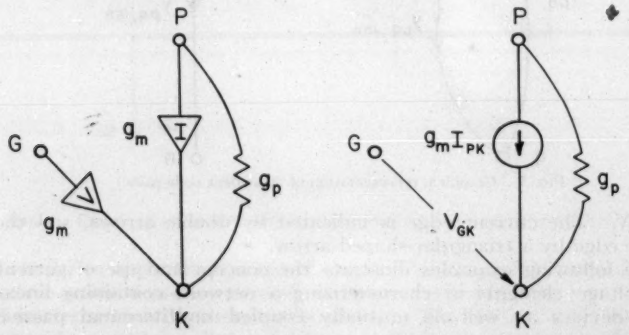


FIG. 4. A triode.

where g_m is the transconductance and g_p the plate-conductance. An equivalent circuit of the triode characterized by Eq. 4 in terms of voltage and current elements is known as the *mathematical* equivalent circuit (5, 6) and is shown in Fig. 5*a*. Figure 5*b* shows the conventional equivalent circuit of a triode for comparison. The graphical representation of Fig. 5*a* is given in Fig. 6.



(a) MATHEMATICAL EQUIVALENT CIRCUIT IN TERMS OF VOLTAGE AND CURRENT ELEMENTS

(b) CONVENTIONAL EQUIVALENT CIRCUIT

FIG. 5. Equivalent circuits of a triode.

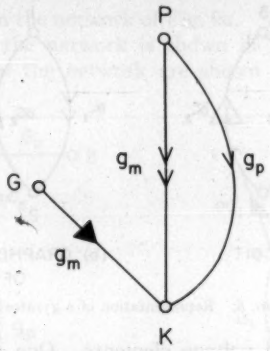


FIG. 6. Graphical representation of Fig. 5*a*.

Example 2. A two-channel gyrator, shown in Fig. 7, may be characterized by the following set of equations (12):

$$I_1 = \alpha_1 V_2 \qquad (5)$$
$$I_2 = -\alpha_2 V_1$$

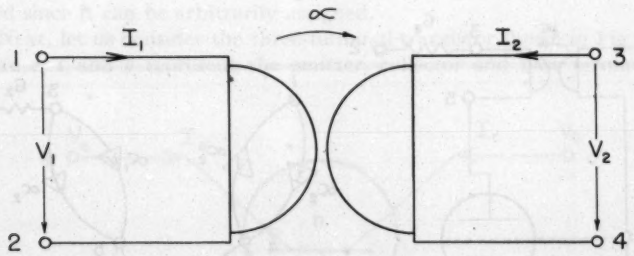
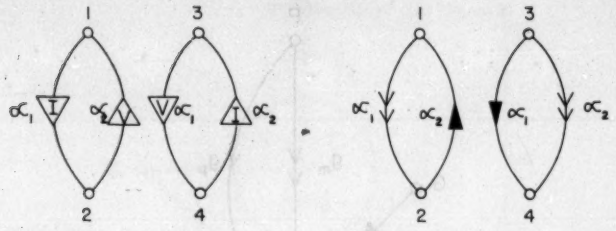


FIG. 7. A gyrator.

where $\alpha_1 = \alpha_2 = \alpha$ is the gyrator admittance.

The mathematical equivalent circuit of the gyrator represented by the terminal characterization of Eq. 5 is shown in Fig. 8*a* and its graphical representation in Fig. 8*b*.

Now a graph representing a device or a network can be decomposed into two subgraphs (7,8). One is the "current graph" containing the edges corresponding to the ordinary elements and current elements. The other is the "voltage graph," with the edges corresponding to the

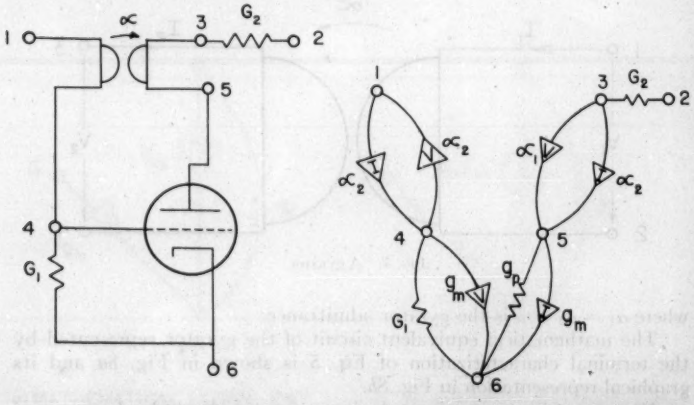


(a) AN EQUIVALENT CIRCUIT OF A GYRATOR

(b) GRAPHICAL REPRESENTATION OF A GYRATOR

FIG. 8. Representation of a gyrator.

ordinary elements and voltage elements. One may immediately see that if a network consists entirely of ordinary elements, the current and voltage graphs of the network are identical; that is, a graph corresponding to an ordinary network is the current graph of the network and at the same time the voltage graph. The orientation of each edge of a graph of an ordinary network is arbitrarily assigned. When a network contains active devices or mutually coupled devices, the current and voltage graphs of the network may not be identical. This is illustrated in Example 3.



(a)

(b) MATHEMATICAL EQUIVALENT OF FIG. 9(a)

FIG. 9.

Example 3. Given the network of Fig. 9*a*. Then the mathematical equivalent circuit of the network is shown in Fig. 9*b*. The current and voltage graphs of the network are shown in Figs. 10*a* and 10*b*,

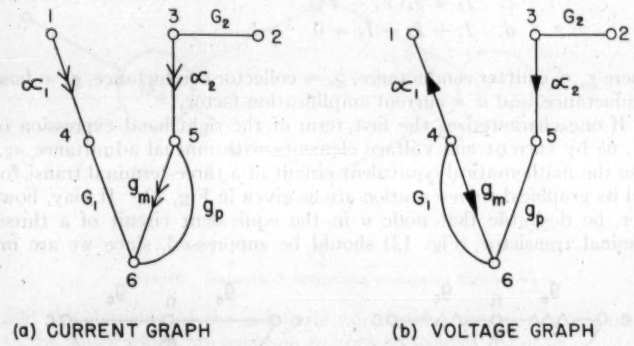


FIG. 10. Current and voltage graphs of Fig. 9*b*.

respectively, where the orientation of the ordinary edges is not indicated since it can be arbitrarily assigned.

Next, let us consider the three-terminal transistor shown in Fig. 11, where *e*, *c* and *b* represent the emitter, collector and base terminals,

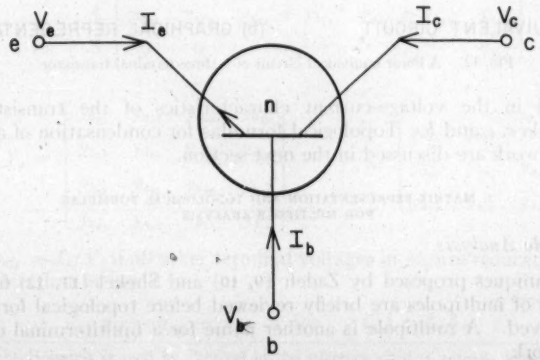


FIG. 11. A three-terminal transistor.

respectively. It may be characterized as a linear active device as follows:

$$\begin{aligned} &a.\quad I_e = g_e(V_e - V_n) \\ &b.\quad I_c = -ag_e(v_e - V_n) + g_c(V_c - V_n) \\ &c.\quad I_b = g_b(V_e - V_n) \\ &d.\quad I_e + I_c + I_b = 0 \end{aligned} \tag{6}$$

where g_e = emitter conductance, g_c = collector conductance, g_b = base conductance, and a = current amplification factor.

If one characterizes the first term of the right-hand expression of Eq. 6*b* by current and voltage elements with mutual admittance ag_e, then the mathematical equivalent circuit of a three-terminal transistor and its graphical representation are as given in Fig. 12. It may, however, be desirable that node n in the equivalent circuit of a three-terminal transistor (Fig. 12) should be suppressed, since we are in-

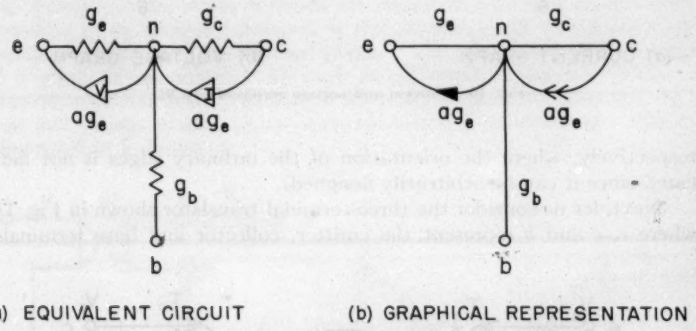


FIG. 12. A linear equivalent circuit of a three-terminal transistor.

terested in the voltage-current characteristics of the transistor at terminals *e*, *c* and *b*. Topological formulas for condensation of a node of a network are discussed in the next section.

2. MATRIX REPRESENTATION AND TOPOLOGICAL FORMULAS FOR MULTIPOLE ANALYSIS

Multipole Analysis

Techniques proposed by Zadeh (9, 10) and Shekel (11, 12) for the analysis of multipoles are briefly reviewed before topological formulas are derived. A multipole is another name for a multiterminal device or network.

Let us consider a linear network N which is a collection of multi-

poles, as shown in Fig. 13. In order to find the voltage-current characteristics of the network N as a five-terminal device, one must first set up the indefinite admittance matrices[4] of multipoles M_a and M_b, and then

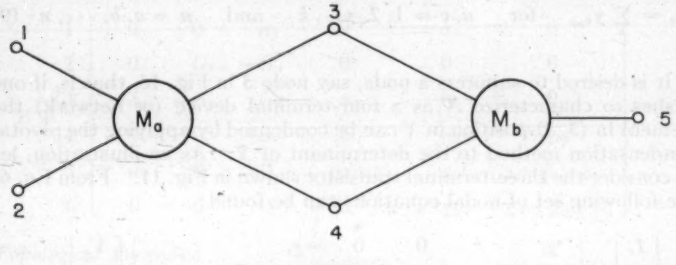


FIG. 13. Network N comprising multipoles M_a and M_b.

add the elements of the indefinite admittance matrices corresponding to the nodes which are common to both multipoles. Let us denote the indefinite admittance matrices of M_a, M_b and N by Y_a, Y_b and Y, respectively. Then Y is found to be

$$Y = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} y_{a_{11}} & y_{a_{12}} & y_{a_{13}} & y_{a_{14}} & 0 \\ y_{a_{21}} & y_{a_{22}} & y_{a_{23}} & y_{a_{24}} & 0 \\ y_{a_{31}} & y_{a_{32}} & y_{a_{33}} + y_{b_{33}} & y_{a_{34}} + y_{b_{34}} & y_{b_{35}} \\ y_{a_{41}} & y_{a_{42}} & y_{a_{43}} + y_{b_{43}} & y_{a_{44}} + y_{b_{44}} & y_{b_{45}} \\ 0 & 0 & y_{b_{53}} & y_{b_{54}} & y_{b_{55}} \end{bmatrix} \end{array} \tag{7}$$

where

$$Y_a = \begin{bmatrix} y_{a_{11}} & y_{a_{12}} & y_{a_{13}} & y_{a_{14}} \\ y_{a_{21}} & y_{a_{22}} & y_{a_{23}} & y_{a_{24}} \\ y_{a_{31}} & y_{a_{32}} & y_{a_{33}} & y_{a_{34}} \\ y_{a_{41}} & y_{a_{42}} & y_{a_{43}} & y_{a_{44}} \end{bmatrix} \qquad Y_b = \begin{bmatrix} y_{b_{33}} & y_{b_{34}} & y_{b_{35}} \\ y_{b_{43}} & y_{b_{44}} & y_{b_{45}} \\ y_{b_{53}} & y_{b_{54}} & y_{b_{55}} \end{bmatrix}$$

and $y_{a_{ij}} = I_i / V_j$ if all other terminal voltages in M_a are reduced to zero.

Note that in Eq. 7

$$y_{ij} = y_{a_{ij}} + y_{b_{ij}} \qquad \text{for} \qquad i, j = 3, 4. \tag{8}$$

[4] The H-matrix defined by Percival in the reference cited is actually identical to the indefinite admittance matrix proposed by Zadeh and Shekel.

Generalizing Eq. 8 for a number of multipoles, M_a, M_b, $\cdots$, M_n, interconnected at terminals 1, 2, $\cdots$, k, the admittance coefficient at each common node is found to be

$$y_{uv} = \sum_{n} y_{n_{uv}} \quad \text{for} \quad u, v = 1, 2, \cdots, k \quad \text{and} \quad n = a, b, \cdots, n \tag{9}$$

If it is desired to suppress a node, say node 3 in Fig. 13, that is, if one wishes to characterize N as a four-terminal device (or network) the element in (3, 3)-position in Y can be condensed by applying the pivotal condensation method to the determinant of Y. As an illustration, let us consider the three-terminal transistor shown in Fig. 11. From Eq. 6, the following set of nodal equations can be found:

$$\begin{bmatrix} I_e \\ I_b \\ I_c \\ 0 \end{bmatrix} = \begin{bmatrix} g_e & 0 & 0 & -g_e \\ 0 & g_b & 0 & -g_b \\ -ag_e & 0 & g_c & ag_e - g_c \\ -g_e(1-a) & -g_b & -g_c & g_b + g_c + g_e(1-a) \end{bmatrix} \begin{bmatrix} V_e \\ V_b \\ V_c \\ V_n \end{bmatrix}. \tag{10}$$

To characterize the transistor as a three-terminal device, the element in (4, 4)-position in the coefficient matrix of (10) is condensed. Thus one gets

$$\begin{bmatrix} I_e \\ I_b \\ I_c \end{bmatrix} = \frac{1}{D} \begin{bmatrix} g_e(g_b+g_c) & -g_b g_e & -g_c g_e \\ -g_b g_e(1-a) & g_b g_e(1-a) + g_b g_c & -g_b g_c \\ -g_e(g_c+ag_b) & -g_b(g_c-ag_e) & g_c(g_b+g_e) \end{bmatrix} \begin{bmatrix} V_e \\ V_b \\ V_c \end{bmatrix} \tag{11}$$

where $D = g_b + g_c + g_e(1 - a)$.

In setting up the indefinite admittance matrix from the mathematical equivalent circuit with the current and voltage elements of a multipole, an element of the indefinite admittance matrix of the multipole, $Y = [y_{ij}]$, is defined as (5, 6)

$$\begin{aligned} y_{ij} = {} & \epsilon \Sigma \text{ admittances of the ordinary edges connected between nodes } i \text{ and } j \\ & + \Sigma \text{ sign} \times (\text{mutual admittance of a corresponding active edge-pair such that the current edge terminates at node } i \text{ and the voltage edge terminates at node } j) \end{aligned} \tag{12}$$

where $\epsilon = 1$ if $i = j$ and $\epsilon = -1$ if $i \neq j$. The sign of the second term is positive if both active edges in a pair are directed away or toward the nodes i and j, respectively, and negative otherwise.

Let us apply the rules stated in Eq. 12 to the equivalent circuit

shown in Fig. 9*b*. Then, the indefinite admittance matrix Y of the network of Fig. 9*a* is found to be

$$Y = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & 0 & \alpha_1 & 0 & -\alpha_1 & 0 \\ 0 & G_2 & -G_2 & 0 & 0 & 0 \\ -\alpha_2 & -G_2 & G_2 & \alpha_2 & 0 & 0 \\ 0 & 0 & -\alpha_1 & G_1 & \alpha_1 & -G_1 \\ \alpha_2 & 0 & 0 & g_m-\alpha_2 & g_p & -g_m-g_p \\ 0 & 0 & 0 & -g_m-G_1 & -g_p & g_m+g_p+G_1 \end{bmatrix} \end{array} \tag{13}$$

Topological Formulas

Let us denote the incidence matrices of a pair of current graphs G_I and voltage graph G_V of a multipole by A_V and A_I, respectively. Y_e denotes the diagonal admittance matrix in which each diagonal element is the admittance of an edge in the graphs. Then, it can be shown that (7, 8) the indefinite admittance matrix of the multipole, Y, is

$$Y = A_I Y_e A_V' \tag{14}$$

where A' is the transpose of A, and the incidence matrix of an oriented graph, $A = [a_{ij}]$, is defined for all nodes and edges in the graph by (13)

$a_{ij} = 1$ if edge j is incident at nodes i and the orientation of the edge is away from node i.
$a_{ij} = -1$ if edge is incident at nodes i and the orientation of the edge is toward node i.
$a_{ij} = 0$ if edge is not incident at node i.

If we denote a submatrix of an incidence matrix A by A_1, which is obtained from A by deleting a row corresponding to the reference node of the network, the node-determinant Δ of the network N is given by

$$\Delta = |A_{1I} Y_e A_{1V}'|. \tag{15}$$

Example 4. Given an ideal three-channel circulator as shown in Fig. 14. The circulator is characterized by

$$\begin{aligned} I_1 &= \alpha_1(V_3 - V_2) \\ I_2 &= \alpha_2(V_1 - V_3) + y_2 V_2 \\ I_3 &= \alpha_3(V_2 - V_1) + y_3 V_3 \end{aligned} \tag{16}$$

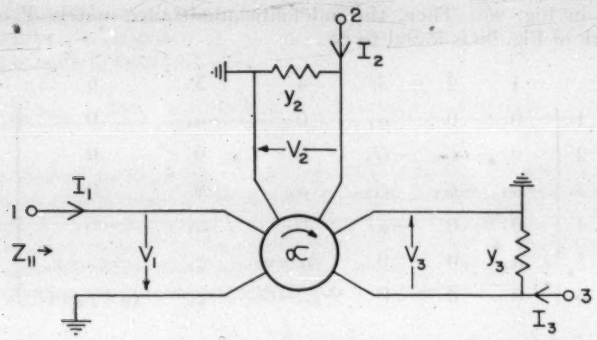


FIG. 14. A three-channel circulator.

where $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$ is the circulator admittance. The mathematical equivalent circuit and its current and voltage graphs are given in Fig. 15, where node n is the reference node and the orientation of the

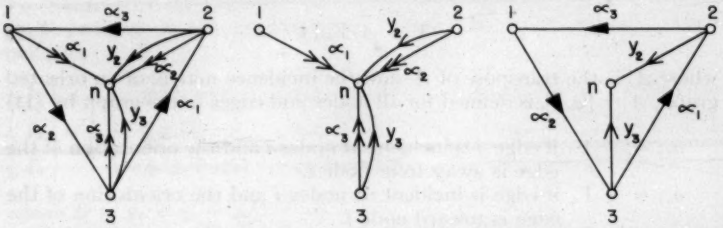


FIG. 15. Graphical representation of the circulator of Fig. 14.

ordinary edges is arbitrarily assigned. The node-determinant of the circulator, Δ, is given as

$$\Delta = |A_{1_I} Y_e A_{1_V}'| = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & -\alpha_1 & \alpha_1 \\ \alpha_2 & y_2 & -\alpha_1 \\ -\alpha_3 & \alpha_3 & y_3 \end{bmatrix} \end{array}$$

and

$$A_{1_I} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccccc} \alpha_1 & \alpha_2 & \alpha_3 & y_2 & y_3 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix} \end{array} \qquad A_{1_V} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccccc} \alpha_1 & \alpha_2 & \alpha_3 & y_2 & y_3 \end{array} \\ \begin{bmatrix} 0 & 1 & -1 & 0 & 0 \\ -1 & 0 & 1 & 1 & 0 \\ 1 & -1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$Y_s = \begin{bmatrix} \alpha_1 & 0 & 0 & 0 & 0 \\ 0 & \alpha_2 & 0 & 0 & 0 \\ 0 & 0 & \alpha_3 & 0 & 0 \\ 0 & 0 & 0 & y_2 & 0 \\ 0 & 0 & 0 & 0 & y_3 \end{bmatrix}. \tag{18}$$

From the Binet-Cauchy theorem and the property of the incidence matrix of a connected graph (7, 8, 13) it is well-known that

$$\begin{aligned} \Delta &= \text{sum of the product of the corresponding majors of } (A_{I_1}, Y_s) \text{ and } A_{V_1} \\ &= \sum_i \epsilon_i \times (\text{complete tree-product } t_i \text{ of current graph } G_I \text{ and} \\ &\qquad \text{voltage graph } G_V \text{ of network } N) \end{aligned} \tag{19}$$

where a complete tree is a tree which is common to both current and voltage graphs, and a tree-product is the product of admittances of a set of edges constituting a tree. ϵ_i is the sign of the complete tree-product t_i taking the value of 1 or -1.

When a network contains only the ordinary edges, the current and voltage graphs of the network are identical, that is, $A_I = A_V$. Therefore the sign of a tree-product is always positive (14). However, when a network includes dependent node-pairs, its current and voltage graphs are different because of active edges in the graphs. Hence, a tree-product which is common to both current and voltage graphs may *not* always be positive. In order to propose a method of sign-determination of a complete tree-product, it may first be necessary to introduce a number of concepts dealing with active edges.

Let us consider a complete tree consisting of edges $(e_1, e_2, \cdots, e_{n-1})$ of a network of n nodes. The tree constituted by the same set of edges in a current graph is called a "current tree" and the tree in the corresponding voltage graph is called a "voltage tree." A pair of active edges (current and voltage edges) included in the corresponding current and voltage trees is called an "active edge-pair" of the complete tree. We shall label the two end-points of current and voltage edges in a pair (i, j) and (m, n). The nodes are chosen such that node i is located

on the side farther than node j from the reference node in the current graph, and node m on the side farther than node n from the reference node of the corresponding voltage graph. Nodes i and m are called "principal nodes" (7, 8) and nodes j and n are called "minor nodes" of the current and voltage edges, respectively. (This is always possible since a tree contains no loops but is connected.) Then, the sign of an active edge-pair is defined by

$$\text{Sign of active edge-pair } y_k = \begin{cases} 1, & \text{if both current and voltage edges in the pair are directed away or toward their minor nodes} \\ -1, & \text{otherwise} \end{cases} \tag{20}$$

Finally, the sign of t_i, the complete tree product, ϵ_i, is determined by

$$\epsilon_i = (-1)^{\gamma} \prod_k (\text{sign of active edge-pair } y_k \text{ in } t_i) \tag{21}$$

where k is the number of active edges contained in the complete tree-product t_i, and γ is the number of interchanges needed to give all active edge-pairs in the complete tree the same principal nodes. The proof for this method of sign-determination is somewhat lengthy and will be found elsewhere (7, 15).

Example 5. Referring to the network shown in Example 3, the node-determinant of the network, with node 6 as the reference node, is the sum of the complete tree-products of the current and voltage graphs shown in Figs. 10*a* and *b*, respectively. It is clear that there exists only one tree which is common to both the current and voltage graphs. This is shown in Fig. 16.

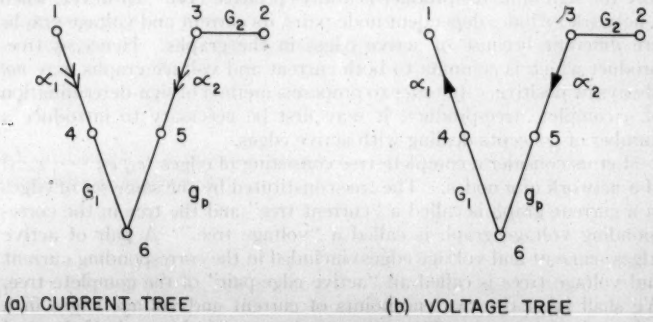


FIG. 16. Complete tree of Fig. 10.

In order to determine the sign associated with the complete tree-product, let us first find the sign of all active edge-pairs in the complete tree. There are two active edge-pairs, α_1 and α_2 as shown in Fig. 17.

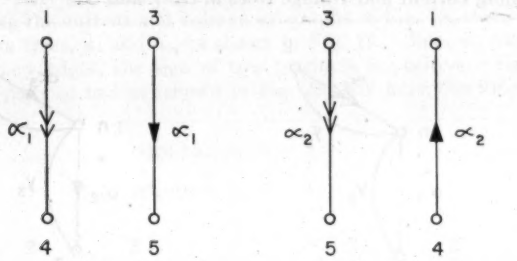


FIG. 17. Active edge-pairs in Fig. 16.

Applying the rules stated in (20) for each active edge-pair shown in Fig. 17, one finds that

$$\text{the sign of active edge-pair } \alpha_1 = 1$$

and

$$\text{the sign of active edge-pair } \alpha_2 = -1$$

One also sees that one interchange is necessary to give the principal nodes of the current and voltage edges in the pair α_2 with the same nodes as in the pair α_1. Therefore, the sign of the complete tree-product, ϵ, is found as

$$\epsilon = (-1)^1(1)(-1) = 1.$$

Hence, the node-determinant, Δ, of the network should be evaluated as

$$\Delta = \alpha_1\alpha_2 g_p G_1 G_2.$$

Generalizing (19) into a cofactor of (p, q)-position, Δ_{pq}, of the node-determinant, Δ, of a network, one obtains

$$\Delta_{pq} = \sum \epsilon_i' \times (\text{a complete tree-product } u_i \text{ of current graph } G_I^{(pr)} \text{ and the corresponding voltage graph } G_V^{(qr)}) \qquad (22)$$

where $G_I^{(pr)}$ is the subgraph of current graph G_I of the network derived from G with node p and the reference node r made coincident, and

$G_V^{(qr)}$ is the subgraph of the corresponding voltage graph G_V with node p and the reference node made coincident. The sign for a complete tree u_i, ϵ_i', is determined by the rules stated in (21) for the corresponding current and voltage trees in $G_I^{(pr)}$ and $G_V^{(qr)}$.

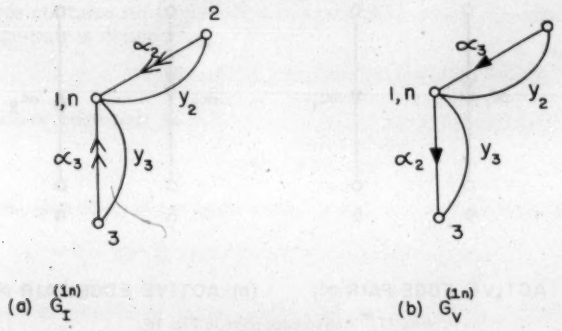


FIG. 18. Subgraphs of Figs. 15*b* and *c*.

Example 6. Find the driving-point impedance Z_{11} at channel 2 of the circulator shown in Fig. 14 for $y_2 = y_3 = \alpha$.

The node-determinant of the circulators, Δ, is evaluated either from (17) or from (19), (20) and (21), as

$$\Delta = \alpha_1\alpha_3 y_2 + \alpha_1\alpha_2 y_3 = 2\alpha^3. \tag{23}$$

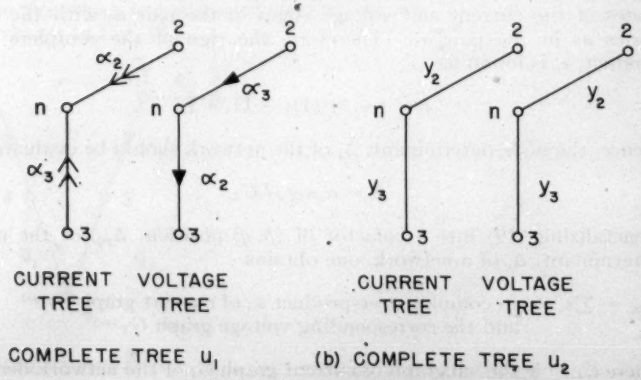


FIG. 19. Complete trees of $G_I^{(1n)}$ and $G_V^{(1n)}$.

In order to evaluate the cofactor of the node-determinant, Δ_{11}, by the topological method proposed in (22), we must first obtain the subgraphs of current and voltage graphs of the circulator (Figs. 15*b* and *c*), $G_I^{(1n)}$ and $G_V^{(1n)}$. They are shown in Fig. 18.

Inspecting the current and voltage subgraphs of Fig. 18, there exist two complete trees, u_1 and u_2, as shown in Fig. 19. Since u_2 consists of the ordinary edges, the sign of tree products is positive. For u_1, active edge-pairs α_2 and α_3 (shown in Fig. 20) will have the following signs:

edge-pair $\alpha_2 = -1$

edge-pair $\alpha_3 = 1.$

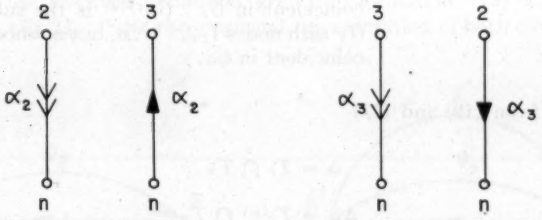


FIG. 20. Active edge-pairs in Fig. 19*a*.

If we interchange the principal nodes of the edges in the pair α_3 only once, they will be the same as the principal nodes of the edges in the pair α_2. Therefore, the sign associated with the complete tree-product of Fig. 19*a*, ϵ_1', is found to be

$$\epsilon_1' = (-1)^1(-1)(1) = 1.$$

Hence,

$$\Delta_{11} = \alpha_1\alpha_2 + y_2y_3 = 2\alpha^2$$

and

$$Z_{11} = \frac{\Delta_{11}}{\Delta} = \frac{1}{\alpha}.$$

Node Condensation

In order to derive topological formulas for the condensation of a node in a network, the following notation is necessary (**14, 16**):

$T_I \overset{\epsilon}{\bigcap} T_V$ = Sum of all complete tree-products of current and voltage graphs, G_I and G_V, of a network N, with n nodes including the sign of each complete tree-product, ϵ, determined by (20) and (21).

$T_I^{(ij)} \overset{\epsilon}{\bigcap} T^{(mn)}$ = Sum of complete tree-products of $G_I^{(ij)}$ and $G_V^{(mn)}$.

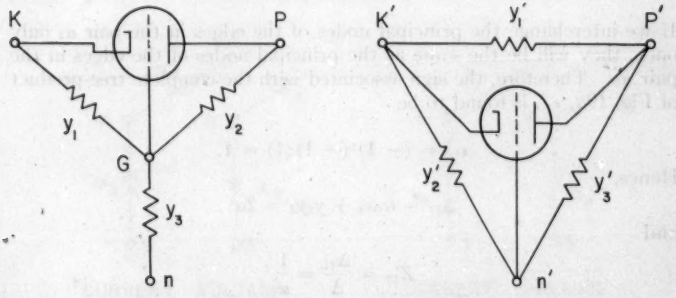
$T_I^{<n,i>} \overset{\epsilon}{\bigcap} T_V^{<n,j>}$ = Sum of all complete tree-products of $G_I^{<n,i>}$ and $G_V^{<n,j>}$. $G_I^{<n,i>}$ is the subgraph of G_I with nodes 1, 2, $\cdots$, n, but not node i, made coincident in G_I. $G_V^{<n,j>}$ is the subgraph of G_V with nodes 1, 2, $\cdots$, n, but not node j made coincident in G_V.

Then, from (19) and (22)

$$\Delta = T_I \overset{\epsilon}{\bigcap} T_V \qquad (24)$$

$$\Delta_{pq} = T_I^{(pr)} \overset{\epsilon}{\bigcap} T_V^{(qr)}$$

where node r is the reference node of the network.



(a) WYE CONNECTED CIRCUIT N (b) DELTA CONNECTED CIRCUIT N

FIG. 21. Common grid transformation.

Let us assume that node n is to be condensed in the network N or its graphical representation G. Then, the indefinite admittance matrix of the condensed network, $Y^* = [y_{\alpha,\beta}{}^*]$, is found to be (16)

$$y_{\alpha\beta}{}^* = \epsilon \cdot 1/y_{nn} \left[T_I{}^{<n-1,\alpha>} \overset{\epsilon}{\cap} T_V{}^{<n-1,\beta>} \right] \tag{25}$$

where y_{nn} is the element in (n, n)-position of the indefinite admittance matrix of the network, and is determined by (12). $\epsilon = 1$ if $i = j$ and $\epsilon = -1$ if $i \neq j$.

Example 7. The wye-delta transformation of tube circuits will be investigated in terms of graphical formulas proposed in (25). Let us consider the so-called "common grid transformation" (3) which is shown in Fig. 21. First the graphical representation of both circuits is

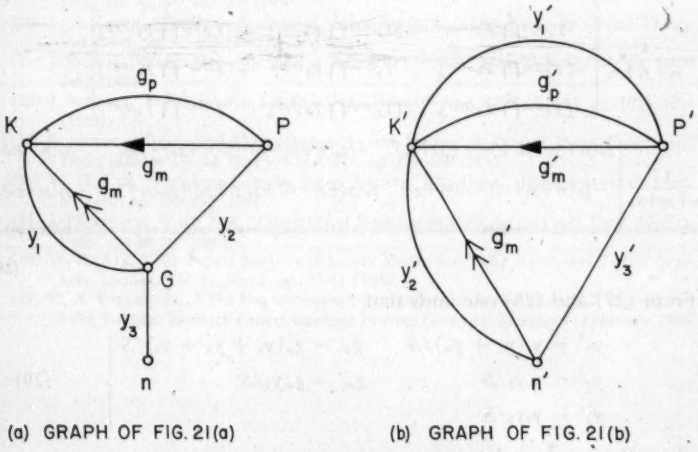


FIG. 22. Graphs of tube circuits of Figs. 21*a* and *b*.

found. This is shown in Fig. 22, where g_m, g_m' are the transconductances and g_p and g_p' the place-conductances of the tubes in Figs. 21*a* and *b*, respectively.

The indefinite admittance matrices of networks N and N^*, Y and Y^*, are found by (12) or by the method proposed by Zadeh (9, 10) as

$$Y = \begin{array}{c} \\ P \\ K \\ n \\ G \end{array} \begin{array}{c} \begin{array}{cccc} P & K & n & G \end{array} \\ \begin{bmatrix} g_p+y_2 & -g_p-g_m & 0 & -y_2+g_m \\ -g_p & g_p+y_1+g_m & 0 & -y_1-g_m \\ 0 & 0 & y_3 & -y_3 \\ -y_2 & -y_1 & -y_3 & y_1+y_2+y_3 \end{bmatrix} \end{array} \tag{26}$$

$$Y^* = \begin{array}{c} \\ P' \\ K' \\ n' \end{array} \begin{array}{c} \begin{array}{ccc} P' & K' & n' \end{array} \\ \begin{bmatrix} g_p'+y_1'+y_3' & -g_p'-y_1'-g_m' & -y_3'+g_m' \\ -g_p'-y_1' & g_p'+y_1'+y_2'+g_m' & -y_2'-g_m' \\ -y_3' & -y_2' & y_2'+y_3' \end{bmatrix} \end{array} \tag{27}$$

By applying (25), Y^* is found, in terms of current and voltage graphs and their subgraphs of N, to be

$$Y^* = \frac{1}{y_1+y_2+y_3} \begin{bmatrix} T_I^{(Kn)} \overset{\cdot}{\cap} T_V^{(Kn)}, & -[T_I^{(Kn)} \overset{\cdot}{\cap} T_V^{(Pn)}], & -[T_I^{(Kn)} \overset{\cdot}{\cap} T_V^{(PK)}] \\ -[T_I^{(Pn)} \overset{\cdot}{\cap} T_V^{(Kn)}], & T_I^{(Pn)} \overset{\cdot}{\cap} T_V^{(Pn)}, & -[T^{(Pn)} \overset{\cdot}{\cap} T_V^{(PK)}] \\ -[T_I^{(Pn)} \overset{\cdot}{\cap} T_V^{(Kn)}], & -[T_I^{(PK)} \overset{\cdot}{\cap} T_V^{(Pn)}], & T^{(PK)} \overset{\cdot}{\cap} T_V^{(PK)} \end{bmatrix}$$

$$= \frac{1}{y_1+y_2+y_3} \begin{bmatrix} g_p(y_1+y_2+y_3)+y_2(y_1+y_3+g_m) & -g_p(y_1+y_2+y_3)-y_2(y_1+g_m)-y_3g_m & -y_3(y_2-g_m) \\ -g_p(y_1+y_2+y_3)-y_2(y_1+g_m) & g_p(y_1+y_2+y_3)+y_1(y_2+y_3)+g_m(y_2+y_3) & -y_3(y_1+g_m) \\ -y_2y_3 & -y_1y_3 & y_3(y_1+y_2) \end{bmatrix} \tag{28}$$

From (27) and (28) one finds that

$$\left.\begin{array}{ll} y_1' = y_2(y_1+g_m)/S & g_p' = g_p(y_1+y_2+y_3)/S \\ y_2' = y_1y_3/S & g_m' = g_my_3/S \\ y_3' = y_2y_3/S & \end{array}\right\} \tag{29}$$

where $S = y_1 + y_2 + y_3$.

CONCLUSION

When a network contains linear multiterminal devices, both active and passive, the theory of linear graphs is always applicable as long as there exists the mathematical equivalent circuit in terms of current and voltage elements. This opens a possibility of analyzing an active and mutually coupled network by a digital computer. The concept of the mathematical equivalent circuit of a network having dependent node-pairs is found to be useful also in the analysis of a magnetic amplifier (17).

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NATIONAL BUREAU OF STANDARDS NEWS

DETERMINING THE ORDER OF CHEMICAL REACTIONS

A simple, rapid mathematical method[1] for determining the order of a chemical reaction has been devised by J. H. Flynn of the National Bureau of Standards. When the order of a chemical reaction is known, the factors influencing the speed of the reaction can usually be determined. Thus, the method promises to be particularly useful to chemical engineers, industrial chemists, and other scientists concerned with controlling rates of reaction. The procedure may also be utilized to detect errors in initial conditions and stoichiometry; and in some cases it provides clues to the forms of complicated kinetic expressions.

When several organic compounds are mixed, many different reactions may take place. To obtain a good yield of any single substance, the temperature, concentration, and other conditions must be controlled so that one specific reaction will predominate. Information on the rates of the various intermediate reactions is of considerable value in establishing these conditions.

In many cases the rate of a chemical reaction is determined by the concentration of the reacting substances. When the rate is directly proportional to the concentration of a single substance, the reaction is said to be of the first order. When the rate is proportional to the concentration of each of two reacting substances, then it is second order, and so forth. Zero-order reactions are these whose rates are influenced by some limiting factor other than concentration.

Although distinct first-, second-, third-, and zero-reactions can be defined in simple terms, very few reactions actually follow a constant order throughout an entire series of steps. In fact, most chemical processes are complex, involving two or more simultaneous reactions.

Because of the complex nature of such reactions, the mathematical description of the over-all process is the result of several rate expressions and consequently is difficult to obtain accurately. Although the Bureau method does not solve all the problems involved, it does simplify the fitting of data to mathematical expressions and provides information on the actual equation. In some cases, it permits interpreting deviations from constant order which are caused by errors in initial times and concentration, and by the reaction following a more complicated rate expression.

The Bureau method of finding order is based on time-ratio tables which were calculated by the Bureau. For a given reaction, values were determined of the time t_x required for the reaction to reach x-per cent completion. Successive overlapping ratio values (t_{15}/t_5, $t_{20}/t_{10}\cdots t_{95}/t_{85}$) were then tabulated as a function of order to give the tables. Time-ratio values can be plotted as

[1] For further technical information, see "Rapid Determination of the Order of Chemical Reactions by Time-Ratios Tables," by J. H. Flynn, Tech. Note 62, which may be obtained from the Department of Commerce, Office of Technical Services, Washington 25, D. C. (75 cents); See also J. H. FLYNN, note in *J. Phys. Chem.*, Vol. 61, p. 11 (1957).

a function of order, as is done for several time-ratios. By using such a family of curves, fractional orders can be easily obtained.

To find the order of a chemical reaction by the Bureau's method, the degree of advancement of the reaction, that is, the extent of the reaction completed, is obtained experimentally at specific intervals. This is done by measuring some physical or chemical property which is a known function of the concentration of a reactant or product. By plotting the different percentages of completion as a function of time, and drawing a smooth curve through these points, values of the times for various degrees of advancement can be obtained. Then, by comparing different ratios of these times to the time-ratio values on the Bureau table, the related order can be rapidly determined.

Not only because of its simplicity, but also because it involves only one subjective step—fitting the smooth curve to time *vs.* concentration data—the method should prove to be particularly valuable to chemists without highly developed mathematic backgrounds. It has some of the advantages of the more cumbersome "differential" expressions often used in kinetic analyses, in that it can be used to interpret deviations in the order. If the order does not follow a simple kinetic expression, the method may aid in recognizing this and in interpreting the correct expressions.

An induction period or initial period of rapid reaction, which may be too short to be recognized by other methods of kinetic analysis, will appear as an error in the initial time or concentration. To distinguish between an error in initial time and an induction period, a new degree of advancement can be calculated from an arbitrary point on the curve representing the concentration as a function of the time.

From the changes in apparent order for successive time ratios, approximate values for the parameters of a much more general type of differential rate equation can be obtained. This equation is applicable to data describing some reversible and simultaneous reactions as well as many catalytic and chain reactions and may prove useful in preliminary kinetic investigations of a chemical system.

REDETERMINATION OF THE HALF LIFE OF CARBON 14

A more accurate value for the half life of carbon 14—important in geological and archeological dating—has recently been obtained by the National Bureau of Standards. The new value is 5760 years, as compared with the previously accepted value of 5568 years.

In performing the redetermination, W. B. Mann and W. F. Marlow of the Bureau's radioactivity laboratory quantitatively diluted high-specific-activity carbon dioxide for counting in length-compensated internal gas counters in the Geiger and proportional regions. Mass spectrometric analyses of parts of the undiluted gas sample were made by E. E. Hughes, R. M. Reese, and V. H. Dibeler to determine the isotopic abundance of carbon 14, which was found to be approximately 44 atoms per cent. Experimental details of the redetermination will be published in the near future. Intercomparative measurements of the isotopic abundance were also carried out with H. W. Wilson of the United Kingdom Atomic Energy Authority.

The redetermined value is in fairly good agreement with the value of

5900 ± 250 years obtained at the Bureau in 1953.[2] The value of 5568 years that has been in use was somewhat arbitrarily chosen. Because of the wide range in measured values of the half life of carbon 14—from 4700 to 7200 years—a weighted average of three values[3] determined by gas counting and by mass spectrometric analysis was tentatively accepted for the purpose of radiocarbon dating of archeological samples. The present measurements have led to the conclusion that the uncertainties in the values obtained experimentally may have arisen almost entirely from adsorption effects.

The Bureau's new value of 5760 years is of particular interest in assessing the age of, for example, the Dead Sea Scrolls. On the basis of the previously accepted half life, these scrolls were found to be 1917 ± 200 years old, that is, dating to about 40 A. D. The new half life will place their age at 1983 years, dating them at about 20 B. C. The change, however, is less than the previously stated experimental error.

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[2] Disintegration rate of carbon-14, by R. S. Caswell, J. M. Brabent, and A. Schwebel, *J. Research NBS*, Vol. 53, p. 27 (1954).

[3] Radiocarbon dating, by Willard F. Libby, p. 34, Univ. Chicago Press, 1955.

BOOK REVIEWS

CONTROL SYSTEM ENGINEERING, edited by William W. Seifert and Carl W. Steeg, Jr. 964 pages, illustrations, 7½ × 10 in. New York, McGraw-Hill, 1960. Price, $15.00.

A large number of books have appeared in recent years on the subject of control systems. This has been in face of a declining rate of advancement in the field and diminishing interest on the part of forward-looking researchers. It is somewhat paradoxical that during the years of rapid advancement of a field, such as occurred for control from the late thirties to the early fifties, few books are written, while when progress has slowed and researchers depart to explore newer areas, we find a plethora of them.

In spite of the many recent books concerned with control theory, *Control System Engineering* is fortunately not "just another book." The authors of this book, and they are eleven in number, have addressed its nearly 1000 pages to the mature graduate student as well as to the practicing engineer. The book is stated to be an outgrowth of a set of lecture notes prepared for a summer course in advanced control theory for engineers from industry. Probably to expedite publication and hold down costs, the book was prepared by a photo-offset process from a typewritten manuscript. This has impaired its readability even though the typing and illustrating have been expertly carried out. The lines are long—nearly fifteen words per line instead of the more common eleven—and the size of the letters is only about three-fourths of standard. In addition, the layout and subtitling are more of the kind one expects from a set of notes than from a book of this size. This reflects on the reader's over-all perspective of the book, as well as on the ease with which he may locate a particular item of interest. These flaws are, of course, common to any book produced in this way.

The first two chapters of the book are concerned with a discussion of mathematical models for control systems. The material is presented well, though it could have been condensed somewhat. The treatment is elementary and directed to a newcomer to the control field. Since this is not true for most of the material which follows, the first two chapters are slightly out of balance with the rest.

The next two chapters deal with ordinary differential equations and with the mathematics of linear systems. The treatment here is unusually thorough for a book on control systems. These chapters will be of considerable reference value to a practicing engineer. A graduate student may, however, prefer the still more comprehensive treatment of books specializing in differential equations or transform theory.

The chapter on basic feedback theory is sketchy. The authors have apparently wanted to avoid competing with other control theory books here. There follow two chapters, one on matrix theory and one on numerical analysis. Both are well written but give their respective topics to light a treatment. The same is true for the chapter on analogue computation.

The chapter on integral equations is very complete; unfortunately however, its connection with control theory is not made sufficiently clear. Nevertheless it is valuable because a knowledge of integral equations is seldom part of a control engineer's repertoire.

There follow three excellent chapters on statistical methods in control theory, concerned with basic theory, engineering applications, and design of optimum systems. These chapters are particularly well written, and they thoroughly cover the subject matter. They can be profitably read by both the graduate student and the practicing engineer, whether or not they have any background in statistics.

The next four chapters are concerned with phase-space analysis, describing functions, time-domain synthesis, and sampled-data systems. They cover the material adequately and can serve both to introduce the material to one unfamiliar with it as well as for reference purposes.

The final chapter on game theory represents a welcome addition to control theory, and the editors are to be commended for recognizing its importance to the control field. The presentation is clear and stimulating.

Not included in the book are linear programming, dynamic programming, multivariable systems, and so-called adaptive systems. Some readers will feel that these modern control theory topics should have been included at the expense of reduced coverage of differential equations, transform theory, and perhaps even integral equations. However, the editors apparently wanted to emphasize the mathematics of control rather than control theory itself.

Summing up, the book contains much that has not previously been presented in control theory books. It suffers from the faults common to all multi-author books of uneven treatment of the subject matter, of occasional inconsistencies in notation, and of a tendency to appear more as a collection of chapters than a cohesive book. It is encyclopedic in its coverage, covering an enormous breadth of material with, unfortunately, an occasional lack of depth. Its use as a textbook is highly questionable; however, as a reference book for the mature graduate student or practicing control engineer it is to be recommended.

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PHYSICS FOR THE INQUIRING MIND. THE METHODS, NATURE AND PHILOSOPHY OF PHYSICAL SCIENCE, by Eric M. Rogers. 778 pages, diagrams, 8¼ × 11 in. Princeton, Princeton University Press, 1960. Price, $8.50.

The author of this book has won a well deserved reputation as an effective lecturer and demonstrator. In opening this unusual and large-sized book one fears that the written word will fall far below the lively quality of the oral presentation. Yet what a pleasant surprise! The clear and humorous hand-drawings immediately seem as though the author were working on the blackboard in front of you, commenting vividly and clearly on the principles illustrated.

The book essentially provides "the general reading, problems and laboratory instructions of a one year course" for undergraduates whose chief field of study lies outside technical physics. While it requires no more mathematics than elements of high school algebra and geometry, it does require "critical reading, good reasoning and clear thinking." It does not claim to be exhaustive but is based on a "block and gap" design. The gaps, where topics are omitted, provide more time for careful teaching of the "blocks," and allow the students to learn by reading and by thinking things out for themselves. The written version of this design contains very few gaps as compared with the actual Princeton course, and thus provides the instructor of the course with a wide choice of blocks to suit his inclination. Elements of the history and philosophy of science are skillfully woven into the text and contribute to improve the understanding and to enlarge the perspective.

One very unusual and most valuable characteristic is the inclusion of demonstrations and of laboratory experiments. The demonstrations are, in general, simple and easy to perform. An example of the historical demonstrations given is Lord Rayleigh's attempt to measure the size of a molecule by measuring indirectly the smallest possible thickness of an oil film covering the water surface. The author helps the reader understand the formation of a monomolecular layer by the analogy of a herd of hungry pigs released near a long food-trough line. As the number of released pigs decreases, they will tend to form what he calls "a monoporcine line." Numerous laboratory experiments and problems are proposed, all intended to stimulate and improve the understanding, and to promote independent thinking.

It would be lengthy and unnecessary to go over the many topics treated. Suffice it to say that the reading and study of this book will prove to be pleasant intellectual adventure for scientists and non-scientists alike. Another interesting feature (at a time when books are so expensive) is the contrast between the low cost of the book in the one hand, and on the other hand its large size and excellent printing.

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The Franklin Institute Laboratories and Temple University

AN INTRODUCTION TO STATISTICAL COMMUNICATION THEORY, by David Middleton. 1140 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1960. Price, $25.00.

Middleton's *Introduction to Statistical Communication Theory* is a book of monumental proportions, but it is impressive and important for other reasons than sheer size. The author, a free-lance consultant, is one of the pioneers in his field; much of the material in this book is a compilation of his own more significant contributions. And the book has been put together with a degree of careful workmanship and attention to detail which is truly awe-inspiring.

Dr. Middleton has divided his book into four parts. Since it is intended (in part) as a graduate-level text, parts one and two—about half the book—are concerned with more or less familiar statistical preliminaries. The first five chapters discuss the elementary properties of random processes and various simple operations on random processes, including an excellent chapter on spectra and correlation functions. Chapter 6 is a short introduction to information theory. The five chapters in part two analyze several special random process models in considerable detail, with particular emphasis, of course, on the normal (gaussian) process and various processes derived therefrom (for example, the envelope and phase at a narrow band normal process).

Part three takes up the second-moment theory of amplitude, phase and frequency modulation and detection, presenting computations of spectra and signal-to-noise ratios for an astonishingly wide range of situations. This section also contains a very brief introduction to the Wiener-Kolmogoroff theory of linear least-mean-square prediction and filtering. Part four is devoted to the statistical theory of signal detection and extraction. Based on the decision theory of Wald as developed by Middleton and Van Meter (*Jour. S.I.A.M.*, Vol. 3, p. 192 (1955) and Vol. 4, p. 86 (1956)), it deals primarily with binary detection systems of the Bayes type. Some new material is included and many examples from radar and communications are considered, with particular emphasis on the weak-signal (threshold) situation.

The mathematical approach throughout is heuristic, and thus the principal appeal of the book would seem to be, as avowed, to "engineers, physicists, and applied mathematicians." On the other hand, Dr. Middleton's earlier publications have frequently frightened off engineering readers by a heavy reliance on analytical formalism and a generally rambling style. Happily, much of the present work represents the author at his best; but still Dr. Middleton definitely remains among those who prefer 1000 words (or even better, 1000 symbols) to any picture.

Moreover, as an introductory graduate-level text at least, the book has several other limitations. The preliminary chapters are a bit sketchy—some knowledge of random signal theory is probably a prerequisite. The balance is uneven—information theory and linear least-mean-square filtering are given relatively short shrift and sequential decision methods are only mentioned. There is a strong tendency to prefer the elegant to the graphic—the Bode-Shannon method, for example, is not even mentioned. And the author is inclined to let his equations and graphs stand for themsevles with only limited attempts to explain what they mean.

On the other hand, there is much that the advanced worker in the field will appreciate. For a wide range of problems, results and methods are presented in elaborate detail. A generally excellent selection of references is provided, both for background and extensions. The indexing and cross-referencing seem adequate. And there are incredibly few typographical errors. Indeed, there seem to be very few errors of any kind; those that this reviewer found were mostly of little substance (although in general we begin to get a bit quarrelsome whenever the author approaches the "real world," either for purposes of modelling or for interpreting the significance of his results; and we suspect that the author could be more careful and consistent in the use of a number of words, such as "realizable," "non-linear," "coherent," "complete set," and "entirely random").

Dr. Middleton is a magnificent manipulator and the last half of his book contains many of his most important results presented in detail. Without doubt, he has made a very useful contribution to the reference shelf.

WILLIAM M. SIEBERT
Massachusetts Institute of Technology

SPACE RESEARCH, edited by H. Kallmann Bijl. 1195 pages, diagrams, 6¼ × 9¼ in. New York, Interscience Publishers, Inc., 1960. Price, $24.00.

When the International Council of Scientific Unions decided to establish the Committee on Space Research popularly called COSPAR, it appeared inevitable that a symposium would be held to permit scientists to present their findings on this facet of the space travel sciences.

Thus in Nice, France, during the second week of January 1960 several hundred scientists from 18 countries presented approximately 100 papers. Dr. H. C. van de Hulst, President of COSPAR, indicated that the organization " . . . will not normally concern itself with such technological problems as propulsion, construction of rockers, guidance and control." Thus the papers are concerned primarily with the progress of scientific investigations carried out by the use of rocket-propelled vehicles.

The approximate 1200 pages of this large volume are divided into seven quite unequal sections.

Part I deals with problems concerning the Earth's atmosphere. One of the most significant papers in the entire volume is the first paper "Pictures of the Earth from High Altitudes and their Meteorological Significance." The films obtained from an Atlas flight permitted correlation with televised pictures to verify that even with the limiting scan lines and noise in the system, the details necessary to study clouds patterns for meteorological purposes are clearly discernible. It is interesting to note at this time that the Tiros meteorological satellite is no longer considered experimental but the satellite is considered an operational one. Some definitive and startling results have come from investigations of the upper atmosphere. H. E. LaGow *et al* indicate that above Fort Churchill at 200 kilometers atmospheric densities vary by a factor of 5! Atmospheric densities here at 50 kilometers are 35 per cent lower in winter than in summer.

Section II deals with the ionosphere. Quite a few of the papers are concerned with the electron density of the upper atmosphere. These densities are determined by the polarization rotation of radio signals from transmitting satellites.

Tracking and telemetering occupy the next part. Here we find J. G. Davies and A. C. B. Lovell's paper "Observations of the Russian Moon Rockets Lunik II and III." The significant part of this paper is disclosed by the words of the authors as: "The variation of frequency with time gives values for the acceleration in the moon's gravitational field which agree closely with that expected for a body falling to the surface at the time when the signals ceased." The authors also say: "No accurate measures of frequency were made on Lunik III, but the direction of the space station was at all times close to the predicted, with the exception of the day following the rocket's passage past the moon, when the error was about 1°." The findings of these researchers should lay to rest those questions which bear on the reality of these lunar probes.

Part IV deals with solar radiation. The availability of rocket probes meant that it was only a matter of time before scientists would propel these to high altitudes above the densest layers of the Earth's atmosphere. Once in this region they could reach the spectrum of the sun below the atmospheric cutoff point. A whole new region was available and scientists have been able to identify the lines. Very early our Naval Research Laboratory personnel concentrated on the study of the solar Lyman Alpha line and quite a few papers in this section explore this. Similarly X-rays and cosmic rays are recognized emissions of the sun and this section explores these. Dr. F. S. Johnson's paper on "The Telluric Hydrogen Corona and Some of Its Consequences" closes the section.

Cosmic radiation represents Section V. Dr. S. F. Singer authors the ranking paper "On the Nature and Origin of the Earth's Radiation Belts" in this section. After a brief historical introduction, he develops the formation of these most important belts circling the Earth. The Russian authors led by S. Sh. Dolginov in their paper "Measuring the Magnetic Fields of the Earth and Moon by Means of Sputnik III and Space Rockets I and II" show how from Space Rocket II magnetometer they determine that if the moon had a magnetic field of over 100 gammas it would have been detected.

Part IV deals with interplanetary dust. One of the most important tasks of the rocket probes and satellites has been the determina-

tion of the distribution and size spectrum of the dust particles in space. This section is devoted to the Russian and American attempts to arrive at definite answers to this problem. The material contained herein indicates that we are still far from a definitive answer. Dr. Harrison Brown's paper seeks to derive the density and mass distribution of meteoroids in the earth vicinity. Dr. John O'Keefe's paper on tektites and the development of the hypothesis of moon origin concludes this section.

The last section of the book deals with the moon and planets. Significant contributions are made by Dr. H. C. Urey on the composition of the moon. Dr. Zenek Kopal's article is of great interest today and the plans for the future of lunar photography are most exciting. The book concludes with two superb contributions by Dr. Joshua Lederberg and Dr. Melvin Calvin and Susan K. Vaughn on extraterrestrial life.

The tremendous spectrum of interests covered by this single volume gives assurance that it will be much in demand and in constant use. The editor is to be congratulated on the magnificent product turned out.

I. M. LEVITT
The Fels Planetarium

OXIDE CERAMICS. PHYSICAL CHEMISTRY AND TECHNOLOGY, by Eugene Ryshkewitch. 472 pages, illustrations, 6 × 9 in. New York, Academic Press Inc., 1960. Price, $16.00.

The book deals principally with ceramics of single phase oxide systems and, as its subtitle clearly suggests, treats them from a physical chemistry viewpoint, discussing the technology from both application and production aspects. Part one (the first 105 pages) is concerned with general fundamentals important in the entire ceramics field and covers particle-size distribution, the physical chemistry of surfaces, workability of oxide masses, sintering processes, firing ceramic materials, and combustion of fuels in furnaces, concluding with a discussion of high temperature furnaces. This portion of the book is an excellent concise discussion of fundamentals and should be useful for newcomers in the field of ceramics and also provides a review of latest trends for those already in the field. The section on high temperature furnaces discusses various types of furnaces used in laboratories as well as in production, and provides the names of commercial suppliers.

The second part of the book is concerned with sintered oxides deliberately excluding silica. One-hundred and forty-seven pages with 245 references deal with all the aspects of alumina from preparation and purification of ores to the production of end products—spark plugs to sintered alumina tool bits. By providing such detailed coverage of alumina, the author was then able to cover the other oxide ceramics by indicating the differences and similarities between each of them and aluminum oxide. Other oxide ceramics covered include spinel $MgAl_2O_4$, magnesia, beryllia, zirconia, zircon, thoria, and ceria. The text is supported with a large number of phase diagrams, which have beens elected by the author after a critical review of the literature.

Throughout the book the author has shown some of the similarities of physical chemical phenomena in oxide systems on the one hand, and metals on the other. A few examples will suffice. The precipitation hardening of magnesia by heat treatment is very similar to the precipitation hardening of certain metal systems. Increasing the hardness of silica by the addition of a softer metal like gold is analogous to adding chromium oxide to corundum to produce the harder material, ruby. Corrosion resistance of a gold silver alloy (1:3) to nitric acid has analogous conditions in oxide systems.

The final chapter pertains to Compound oxide-metal and oxide-carbide ceramics and, while not purely oxide ceramics, was fortunately included because of its great technical importance and the similarity in technology.

The high gloss on the pages was found to be objectionable to the reviewer because of the glare it produced. Admittedly this is less of a problem for one who is using it as a reference than when it is being read in its entirety.

WILLIAM SCHUELE
The Franklin Institute Laboratories

OPTICS OF THE ELECTROMAGNETIC SPECTRUM, by C. L. Andrews. 501 pages, illustrations, 6 × 9 in. Englewood Cliffs (N. J.), Prentice-Hall, Inc., 1960. Price: $12.00 (trade); $9.00 (text).

One of the great achievements of classical physics was to have integrated a large body

of very diverse phenomena, very different in their physical manifestations, into a single family called the electromagnetic spectrum. Although this has been accomplished for more than half a century most elementary or intermediate text books have found it more convenient to treat optics and electromagnetic waves separately, content to show the connections in an appendix or a last chapter. However, after a period of relative neglect, physical optics is experiencing a new "renaissance," due precisely to the unity of features connecting all electromagnetic phenomena. It is this unity that the present text stresses from the outset.

After a brief but well written historical introduction, the author devotes the rest of the first chapter to the generation and detection of electromagnetic waves and to a non-mathematical description of their unity and general features. In the following three chapters, the general properties of waves (propagation, superposition, standing waves) are exposed. The complex notation is used to represent the vibration spiral. Interference phenomena are thoroughly examined with emphasis on Michelson's and Fabry-Perot interferometers. A careful treatment of Fraunhofer and Fresnel diffraction is followed by a general treatment of electromagnetic theory and a study of dispersion. The unity announced in the first chapter is thus shown in detail. However, to this reader one of the crucial connections between electromagnetic theory and light waves occurs through Kirchhoff's equation and method. The latter, however, is used before the treatment of electromagnetic waves and is derived independently in an appendix. The series of approximations leading from the electromagnetic wave equations to Kirchhoff's equations and even to geometrical optics are well worth a treatment, and such a treatment need not be mathematically sophisticated (see, for example, Joos' *Theoretical Physics*). The rest of the book is devoted to the optics of anisotropic media and to optical rotation.

The conventional chapter heads may be misleading because the content is most unconventional. Written in a style fluent and clear, the book abounds in original ideas, in microwave demonstrations bringing the optical phenomena to a macroscopic scale. The line drawings, diagrams and photographs are unusually good, and the significant experimental details are stressed. Except for the slight reservation mentioned above, this reader recommends highly this book as an excellent and up-to-date textbook on the electromagnetic spectrum.

HENRI AMAR
The Franklin Institute Laboratories and Temple University

THE MATHEMATICS OF RADIATIVE TRANSFER, by I. W. Busbridge. 143 pages, 5½ × 8½ in. New York, Cambridge University Press, 1960. Price, $5.00.

The 1934 monograph of E. Hopf dealing with the mathematical methods of radiative equilibrium still remains the basic exposé of radiative transfer problems arising in astrophysics. Since that time, however, (*a*) the nonconservative case was shown to be of very great importance (and Hopf's work deals exclusively with the conservative case); (*b*) neutron diffusion was shown to lead to similar mathematical equations; and (*c*) a few new mathematical methods and techniques were developed especially by Chandrasekhar and Ambartsumian. Hence the need for a more modern and more complete presentation of these problems. In this short Cambridge tract an active contributor to this field undertakes this task.

The book is well written and will prove to be a valuable synthesis for the mathematical physicist and astrophysicist. For others, however, the book will appear too formal. It is regrettable that the physical meaning of the equations and the physical motivations of these mathematical techniques have been neglected.

HENRI AMAR
The Franklin Institute Laboratories and Temple University

THERMOELECTRIC MATERIALS AND DEVICES, edited by Irving B. Cadoff and Edward Miller. 344 pages, diagrams, 6 × 9 in. New York, Reinhold Publishing Corp., 1960. Price, $9.75.

This particular volume is the outgrowth of a one-week course on thermoelectric materials and devices offered by New York University in June, 1959, and again in June, 1960. The lectures were given by scientists and engineers

active in the development of thermoelectric materials or in the design of devices using these materials. Each lecturer was given full liberty to develop his topic in his own way. However desirable this may be in the classroom, it is not a recommended method for constructing a book. The duplication of material and non-uniformity in the use of symbols and units, which should have been eliminated by more forceful editing, makes the way to understanding more difficult than it need be. This is not meant to imply that the individual contributions are not worthwhile in themselves. The topics are well chosen and, individually, well treated. There is certainly advantage to having these papers all under one cover, especially since an index is provided. Those interested in an introduction to thermoelectric materials research will find the volume useful, although those who think a book should be a book will not be happy with it.

F. J. DONAHOE
The Franklin Institute Laboratories

MATTER AND ANTIMATTER, by Maurice Duquesne. 127 pages, plates, $4\frac{1}{4} \times 7\frac{1}{4}$ in. New York, Harper & Brothers, 1960. Price, $1.75 (text).

This is one of two small books which are units of the Science Today series published by Harper & Brothers. Both of the books are translated from the French and both are classics in that they are reviews of one facet of astronomy and one of physics.

In *Matter and Antimatter* Dr. Maurice Duquesne, Director of Research in Nuclear Physics at the Radium Institute in Paris, has reviewed particle physics leading to the discoveries of the antiparticles. The author is most clever in his treatment by beginning with atomic theories familiar to the college student and leading up to the prediction of the antiparticles as the work of the eminent English mathematician Dirac.

The author shows how the introduction of relativistic mechanics resulted in the energy of a particle having both positive and negative values, and in the famous paper of December 6, 1929, Dirac indicated the possibility of electron energies which were negative. At that time he wrote: "Thus an electron with negative energy moves in an external field as though it carries a positive charge." The electron with negative energy we call a positive electron or positron and was found by Anderson in his classic experiment a few years later. While anti-protons and anti-neutrons were suspected for some time as a result of Dirac's pioneering efforts, it was not until 1955 that scientists could begin creating these particles. The principal reason for the long delay in the discovery was the tremendous energies with which the incident proton had to be endowed—some 6 billion electron volts—in order to produce the anti-proton. The machines to accelerate particles to this energy are the product of the past few years. Even the detection of the anti-particles was an achievement of no mean magnitude with consumate skills being used to produce the detectors.

Following the excellent background on the creation and detection of antiparticles the author speculates for just a few brief pages (an altogether too short exposition of this fascinating topic) on the meaning of anti-matter in the universe.

The author has written a fascinating and illuminating book. One point which is most apparent as this review of atomic theory unfolds is the tremendous number of ranking scientists—Nobel prize winners—who participated in the story of the atom.

Dr. Duquesne has made a significant contribution in his latest work.

I. M. LEVITT
The Fels Planetarium

THE WIDER UNIVERSE, by Paul Couderc. 128 pages, plates, $4\frac{1}{4} \times 7\frac{1}{4}$ in. New York, Harper & Brothers, 1960. Price, $1.75 (text).

This is the second of two units of the Science Today series.

The advent of the large telescope has given the astronomer a probe to reach deep into space. With every large telescope came more and more observations which the astronomer has tried to knit into a unified whole. When the large telescopes ceased being built—they simply cost too much—a new tool, radio astronomy, took over to enable the astronomer again to extend his reach and to verify his findings with the optical telescope. This the astronomer has accomplished and the result

is a fascinating story of the evolution of man's ideas about the universe. No one can claim that we know all the answers to the questions posed about the universe but from the way the pieces of the puzzle are falling into place, we can say that our ideas cannot be too far wrong.

In *The Wider Universe* Dr. Paul Couderc, of the Paris Observatory, has developed our ideas about the star systems or galaxies and describes the patterns they possess.

The author begins in a low key dealing with the age of the Earth and swings into a description of our Milky Way. This is described in shape, motion, content and the role that radio astronomy played in delineating the spiral structure of the galaxy.

The question of distance determination to these giant star systems occupies the author and an excellent summary of the principles involved in these determinations is given.

The next logical step is the clustering of the galaxies and the work of both the Mt. Palomar and Lick Observatory personnel is described. It is rather surprising to hear that the Lick experts hold that galaxies exist only in clusters.

In this chapter we find evidence of the tremendous strides in this branch of astronomy. While this book has just been released, some of the numbers quoted are at odds with the latest determinations. The author indicates that we can photograph galaxies at a distance of 1½ billion light years while giant galaxies may be seen even farther away. Actually, pictures and spectra of galaxies at 6 billion light years have been obtained at Mt. Palomar.

As for the spectral shift, the 160 kilometers a second per million light years has been cut down to 55 kilometers per million light years by the reevaluation of our celestial yardstick —the cepheid variables. However, in recent observation of the galaxy at 6 billion light years Mt. Palomar astronomers indicated the velocity of recession to be about *90,000 miles a second*. This would lead to the conclusion that these astronomers are using 25 kilometers per second per million light years instead of 55 kilometers per second.

In the concluding chapter, the author indicates that the expansion of the universe is to be preferred over the other cosmological theories. This, however, must be regarded as a subjective observation.

The Wider Universe is an excellent little book yielding a capsule description of the Universe.

I. M. LEVITT
The Fels Planetarium

ELECTRONIC PROCESSES IN SOLIDS, by P. Aigrain, R. J. Coelho, and G. Ascarelli. 67 pages, diagrams, 6 × 9 in. New York, John Wiley & Sons, Inc.; Cambridge, The Technology Press; 1960. Price, $4.00.

This book is based on lecture notes from a series of eight lectures given by Professor Pierre Aigrain at the Electrical Engineering Department of M.I.T. in May, 1957. A wide program dealing with the theory of electronic processes in solids is covered. The table of contents is impressive and the 15 sections range from the Crystal Hamiltonian and the Born-Oppenheimer approximation to the Electron-Phonon scattering phenomena and High-Field Conduction phenomena.

In view of the limited space (67 pages) and the variety of topics treated, the presentation is unusually compact and formal, but also unusually clear and original. It is these last features that probably justify its publication, since the very popular material treated has been published in various forms in numerous publications during the last few years. Of particular interest are the sections on High-Field Conduction ("hot electrons") and on non-intrinsic scattering mechanisms (that is, by ionized impurities, dislocations, etc.). A short and intelligently selective bibliography terminates the text. These lecture notes of Prof. Aigrain will be read with profit by most workers in this field.

HENRI AMAR
The Franklin Institute Laboratories and Temple University

PUBLICATIONS RECEIVED

ELECTRONICS IN ENGINEERING, by W. Ryland Hill. Second edition, 340 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1961. Price, $8.00.

HYDRODYNAMICS. A STUDY IN LOGIC, FACT, AND SIMILITUDE, by Garrett Birkhoff. Second edition, revised and enlarged, 184 pages, diagrams, 6 × 9 in. Princeton, Princeton University Press, 1960. Price, $6.50.

INTRODUCTION TO ATOMIC AND NUCLEAR PHYSICS, by Otto Oldenberg. Third edition, 380 pages, illustrations, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1961. Price, $7.95.

MATHEMATICAL TABLES. VOLUME 4. TABLES OF WEBER PARABOLIC CYLINDER FUNCTIONS AND OTHER FUNCTIONS FOR LARGE ARGUMENTS, by L. Fox. 40 pages, 8½ × 11 in. London, Her Majesty's Stationery Office, 1960. Price, 12s. 6d. net, by post, 12s. 11d.

THE ATOMIC PROBLEM. A CHALLENGE TO PHYSICISTS AND MATHEMATICIANS, by Lancelot Law Whyte. 56 pages, 5¼ × 7¾ in. London, George Allen and Unwin Ltd., 1961. No price.

FUN WITH ELECTRICITY, by Tom Kennedy, Jr. 128 pages, illustrations, 5½ × 8½ in. New York, Gernsback Library, Inc., 1960. Price: $2.65 (soft cover); $3.75 (hard cover).

SEMICONDUCTOR ABSTRACTS, VOLUME VI, 1958 ISSUE, edited by J. J. Bulloff and C. S. Peet. 527 pages, 8½ × 11 in. New York, John Wiley & Sons, Inc., 1961. Price, $14.00.

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The Secretary of The Franklin Institute will gladly furnish you with additional information. Write to him at The Franklin Institute, Benjamin Franklin Parkway at Twentieth Street, Philadelphia 3, Pennsylvania.

THE FRANKLIN INSTITUTE

MINUTES OF THE STATED MEETING

February 15, 1961

The Stated Meeting of The Franklin Institute was held at 8:15 P.M. in the Lecture Hall. Wynn Laurence LePage, President, called the meeting to order with approximately 220 members and guests present.

The President stated that the minutes of the Stated Meeting of December 21, 1960, were published in the January issue of the JOURNAL. There being no corrections or additions these minutes were approved as published. The minutes of the Annual Meeting of The Institute held on January 18, 1961, will be published in the February issue of the JOURNAL and will be presented for approval at the next Stated Meeting of The Institute.

The President announced that in order to accelerate the Franklin Institute Development Fund effort, a very important dinner is planned for the evening of Wednesday, April 19, 1961, at The Institute, and as this date coincides with the date of the Stated Meeting of The Institute he requested that the members approve the postponement of the Stated Meeting of Wednesday, April 19, 1961 to Wednesday, May 17, 1961. On motion duly made and seconded, the postponement of the Stated Meeting of The Institute of April 19, 1961, to May 17, 1961, was unanimously approved.

The lecture this evening, the President announced, would be the Sixth Annual Philip C. Staples Lecture, established by many friends of the late Mr. Staples in his memory because of his valuable contributions to The Franklin Institute and to the community.

Mr. LePage then introduced the speaker of the evening, Professor L. Don Leet, Seismologist in charge of the Harvard Seismograph Station, whose subjest was "Earthquakes and Tidal Waves."

Professor Leet's most interesting talk on earthquakes and tidal waves, using diagrams, slides and historical facts, was received with great enthusiasm as was evidenced by the interesting question period following the talk. Mr. LePage extended to Professor Leet The Institute's deep appreciation for his outstanding talk on seismology. The meeting was adjourned at 9:35 P.M.

WILLIAM F. JACKSON, JR.
Secretary

COMMITTEE ON SCIENCE AND THE ARTS

(Abstract of Proceedings of Stated Meeting held Wednesday, February 8, 1961)

HALL OF THE COMMITTEE,
PHILADELPHIA, FEBRUARY 8, 1961.

MR. J. PHILIP EVANS in the Chair. The following report was presented for final action:

No. 3260: Impacting—New Forging Process.

This report recommended the award of an Edward Longstreth Medal each to Robert L. Alcorn, Jr., Eugene C. Clarke, and Henry A. Weyer, of Chambersburg, Pennsylvania, "In consideration of their invention of the 'Impacter' method of forging parts and controls therefor which marks a notable advance in the ancient art of forging toward modern utilization of this art by, among other things, substantially eliminating shock and vibration, facilitating high speed operation and by making possible forging of metals at extremely high temperatures."

D. S. FAHRNEY,
Secretary to Committee

NEW BOOKS IN THE FRANKLIN INSTITUTE LIBRARY

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CONGRÈS INTERNATIONAL DE PHYSIQUE NUCLEAIRE, PARIS, 1958. INTERACTIONS NUCLEAIRES AUX BASSES ENERGIES ET STRUCTURE DES NOYAUX, COMPTES RENDUS. Présentés par P. Gugenberger. Dunod, 1959.

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CURRENT TOPICS

Space Environment Simulation Laboratory.—A strange place where ice will melt away amid temperatures of 300 degrees below zero is being constructed for the General Electric Company at historic Valley Forge. To become an important part of the Company's planned Space Technology Center, this $6,000,000 research facility will approximate the alien conditions of space millions of miles from Earth. Not only is this one of the largest space simulators planned or being erected in this country, but the facility will uniquely have "sun" beaming energy at the same intensity and appearing as far away as the real sun. The combination of pressure, solar and space temperature conditions that will be created in the simulator will make it an exceptional space research tool yet to be matched or surpassed in existing or known planned space simulators, according to GE officials.

Called a Space Environment Simulation Laboratory, the facility will be used for testing actual space vehicles and for research leading to the design of these vehicles. The Laboratory is being financed by General Electric.

Many alien conditions found only in space will be duplicated in the simulator. For instance, the walls of the simulator will be entirely dark and cold approximating "cold black space." A simulated sun will be provided which will appear to the test vehicle as a source of energy at infinite distance.

Only when a reflecting object is placed in the path of the sun's rays of energy will there be any light and heat—and then, only on the side facing the sun. The opposite side will be dark and also invisible because of the lack of reflection in the simulator walls.

The pressure in the simulator will be reduced to as low as one billionth of atmospheric pressure (14.7 psi. at sea level). The air in the vessel will be so "thin" that only relatively few molecules will remain in each cubic inch of volume.

Air pressure will be reduced in the simulator by the familiar mechanical pumping method; as well as by the less familiar methods of condensing or freezing of the internal atmosphere on very cold wall panels. The latter involves the pumping of liquid nitrogen and helium gas through wall panels which reduces the wall temperature to as low as 420 degrees below zero. At these temperatures most gases will condense or freeze on the wall panels, reducing the pressure inside the simulator.

In some respects the simulator will operate like a giant refrigerator, and when a layer of the condensed gases is accumulated on the wall panels it will have to be "defrosted." However, General Electric engineers state the simulator can be operated continuously for many months before "defrosting" becomes necessary.

To simulate the radiation from the sun a combination of special General Electric lamps and a unique optical system was designed. Four lamp groups, each containing thirty-seven 5000 watt zenon arc lamps, will be directed at a mirror and the energy reflected into the simulator. Owing

to the optical system, the sun will appear to be 93,000,000 miles away as the real sun is from the earth.

The egg-shaped simulator or pressure vessel will be 32 ft. in diameter and 54 ft. high, enabling the testing of space vehicles as large as 20 ft. and weighing as much as 5 tons. The simulator will be housed in a building 100 ft. square and 88 ft. high. In addition to the simulator, the building will contain service areas, offices, laboratory space, control room, and test vehicle preparation areas.

The simulator will enable General Electric engineers and scientists to test the operation of full-size space vehicles and their equipment in an accessible location and under controlled conditions prior to actual space launchings.

Among some of the unknowns that will be answered by testing in the simulator will be how well a vehicle and its equipment operate in space, the balance of heat in a vehicle, and the behavior of materials in high vacuum.

Use of the simulator will include system testing of solar power systems, manned and unmanned space vehicles, and space suits.

The Space Environment Simulation Laboratory is being erected on a 130-acre site of the General Electric Space Technology Center about 17 miles west of Philadelphia. General Electric plans to make the multi-million dollar Center headquarters for its research and development work in advanced space vehicle programs. It will be ready for limited testing in 1961 and fully activated by mid-1962.

Associative Memory Permits Shortcuts to Stored Data.—A new kind of magnetic core memory for computers which contains built-in "shortcuts" to stored information has been reported by the International Business Machines Corporation. An experimental memory, described in the IBM *Journal of Research and Development*, represents a significant advance in an approach to a relatively new concept called "associative memory." The prototype unit was constructed with existing components to show the feasibility of a ferrite magnetic core associative memory that can operate at speeds equivalent to today's memories.

"Shortcuts" provided in such a memory may, in the future, speed up access to stored information by eliminating the need for a considerable amount of programming in certain kinds of processing. While the experimental unit disclosed today has a capacity of only four information "bits," IBM has carried out extensive studies indicating that a large-scale associative memory, comparable in speed and capacity to conventional units now available, is technically feasible.

In a conventional memory, data are stored in "registers" at specific "addresses." To pluck the word out of the memory, the program being fed into the computer must specify the correct address. The experimental unit IBM has reported is a "fully associative memory." This means that it is organized so that a word can be stored in any register and no address need be given to locate it. Furthermore, any fragment of a stored word may be used as an interrogation of all the memory registers simultaneously.

IBM research engineers put it this way: "You tell the memory what it is you know about the subject you're interested in, and the memory will respond by supplying all that it knows relating to the subject."

The engineers state that the experi-

mental associative memory can also be operated as an ordinary addressable memory upon command and without any sacrifice in capability. They emphasize that the IBM associative memory approach aims at great improvements in computer system speed and flexibility through more advanced design rather than through new kinds of components alone. Although the experimental unit is a magnetic one, the associative concept is not limited to magnetic units but may be used in other memories as well. Previous technical papers by research engineers of IBM's Data Systems Division have indicated that associative memory concepts may be used in cryogenic memories.

To illustrate how an associative memory system might be used, the following customer problem is described in the IBM *Journal* article: Suppose a customer requires information on a stock of electrical switches. Data on each item specifying its number, switch size, current capacity, voltage and other information will be stored in the memory. If the customer desires a switch of a given current capacity and voltage, a simultaneous interrogation of all memory registers for these values will produce information on all stored items which fit these requirements.

Membrane Permeation Cell.—A new laboratory-scale membrane permeation cell incorporating recent developments in using selectively permeable membranes for changing the composition and separating components of fluid mixtures is available from Ionics, Incorporated, Cambridge 42, Mass.

Membrane permeation is a new unit operation utilizing membranes which permit easier passage of one or more of the components of a mixture relative to the remaining component. Appropriate selection of membranes and operating conditions such as temperature and pressure make possible the effective separation, for commercial purposes, of various components in industrial process streams.

Typical of the many applications which lend themselves to membrane permeation techniques are the breaking of azeotropes, separation of close-boiling mixtures, removal of water from organics and driving equilibrium reactions to completion. According to Ionics, the new permeation cell makes possible complete process research investigations of promising commercial applications in the customer's laboratory, including economic evaluations, determination of process variables and membrane selection and development programs leading to optimum pilot plant designs and full-scale commercial processing cost estimations.

In Ionics' standard cell all metal parts in contact with fluids to be tested are made of stainless steel. Other materials can be furnished upon request.

The cell has a nominal volume of 0.4 gallon (1, 55 liters) and an effective membrane area of 0.165 sq. ft. (23.8 sq. in.; 153, 3 sq. cm.). Standard rated working pressure is 150 psig (11, 6 kg./sq. cm. abs.) at 325° F. (163° C.). Higher pressure ratings can be furnished upon request.

The cell is equipped with a heating mantle, pressure gauge, thermometer well, and may be operated on a batch or continuous basis. An internal chamber provides a means for rapid cooling. If desired, the chamber also may be used as an alternative heating or temperature control system. A variable-speed stirrer promotes uniform temperature and composition throughout the charge.

Optionally available accessories in-

clude a vacuum pump, condensers, glassware, etc., and permit "plug-in" operation of the cell.

Radio Telescopes Launched into Space.—Prof. Donald H. Menzel, Director of the Harvard College Observatory, announced that the slender, Blue Scout rocket, launched from Cape Canaveral on Jan. 7, lifted radio telescopes above the Earth's opaque ionosphere for the first time.

As the rocket arced above the ionosphere, four small radio telescopes broadcast back to Earth measurements of low-frequency cosmic radio noise. The ionosphere of the Earth blocks passage of the low-frequency cosmic radio spectrum to terrestrial receivers.

Detailed studies with space radio telescopes will permit astronomers to gather new information about cosmic radiations that originated thousands of light years away, the weak magnetic fields in interstellar space, the proton and electron clouds between the sun and the Earth, and the radio noise storms on Jupiter.

Prof. Edward Lilley, who heads the Harvard space radio telescope project, said that a preliminary study indicated all four radio telescopes had operated successfully throughout the flight in space.

Each radio telescope was composed of a transistorized receiver, the size of a pocket radio, and a collapsed antenna which extends on command to a length of ten feet. The total weight of the four radio telescopes was less than ten pounds. They operated at frequencies between 700 kilocycles and 13 megacycles.

The radio telescopes in the Blue Scout rocket were also used to conduct a new measurement on the Earth's ionosphere. Harvard scientists G. Huguenin, H. Gravel, and F. Jandrue, with operations centered at San Salvador Island, sent high-powered, coded radio pulses toward the ionosphere during the rocket's flight. Most low-frequency radiation is reflected back to the Earth by the ionosphere. Scientists have suspected, however, that the ionospheric shell may have small "holes," through which a small fraction of the terrestrial radio energy could leak. With the combination of the space radio telescopes and the San Salvador transmitters, the Harvard group has a new technique for studying the ionosphere and the leakage of terrestrial radiation into space.

Living Insects on High Antarctic Plateau.—Living insects and mites have been found at an elevation of 6000 ft. above sea level about 90 miles from McMurdo Sound, Antarctica, the National Science Foundation reports. This is believed to be the highest altitude at which insect life has ever been encountered on this frigid, ice-covered continent.

The organisms were found by Keith A. J. Wise, a New Zealander working for the Bernice P. Bishop Museum in Honolulu. Wise is one of four Bishop Museum scientists currently studying the distribution and dispersal of airborne and ground organisms, primarily insects, in the Antarctic. The work is supported by a grant from the National Science Foundation through its U. S. Antarctic Research Program (USARP).

While on a field trip to the Mt. Gran area at the head of the Mackay Glacier with other USARP scientists, Wise came across approximately two dozen collembola or springtails—an order of primitive wingless insects—under loose rocks on the snow-free surface of the plateau. He also found about one dozen free-living, non-parasitic mites in the same general area.

(Mites are not insects, having four rather than three pairs of legs in the mature stage, but both insects and mites are arthropids, belonging to the *Phylum arthropoda.*) No plant life of any kind was found.

Wise believes the altitude at which he encountered the springtails and mites to be the highest at which these tiny forms of life have ever been collected in the Antarctic. Wise noted that the specimens of springtails taken were all white, whereas specimens he encountered a month earlier at Hallett Station, about 300 miles further north, were all black. Later specimens taken at the base of the glacier, at sea level, were all black, while still other specimens seen at Mt. Suess, between the other two collection sites, were of both white and black varieties. While the exact significance of these color differences is not yet known, it appears to indicate the existence of two different distribution areas and a possible boundary between them.

Terrestrial arthropods reported to occur in the Antarctic include collembola, flies, sucking and biting lice, mites, and ticks. About fifty species of these have been collected so far, of which about half are parasitic and the remainder free living.

In general, sufficient exposure to the sun, moisture, food in the form of algae and moss, and reasonable protection from the wind are the conditions vital to the support of insect life on this continent. Data and specimens collected in this area and in the Palmer Peninsula area will be forwarded to the Bishop Museum in Honolulu, where they will be further evaluated.

The current NSF-supported study may provide data that will assist meteorologists, botanists, microbiologists, agriculturalists, and quarantine experts. Data collected may shed light on climates, air currents and sea currents of the past and present, as well as on the actual population and evolution of arthropods on oceanic islands and within the Antarctic.

SPAT Transistors.—Philco Corporation of Lansdale, Pa., has announced that a major upgrading in the reliability of medium frequency silicon transistors has been achieved with a new production process which has made possible the development of the Silicon Precision Alloy Transistor (SPAT).

H. K. Ishler, director of semiconductor operations at the firm's Lansdale Division, explained that the new SPAT PNP silicon transistors are produced by a unique strip alloy technique, making possible unprecedented product uniformity. Strip alloying permits an accurate measurement of the diode voltage and beta of every SPAT transistor during the manufacturing process.

In the strip alloying process, each transistor is indexed on a precision carrier which guides the device into a minified alloying furnace and positions it in perfect alignment with the heat source. In this manner every transistor is heat-treated identically.

Previously, several transistors placed in carbon boats were carried into a large alloying furnace by means of a conveyor belt. In this "batch" process heat cannot be uniformly applied to each transistor, thus making it impossible to accurately direct or even know the exact conditions of heat concentration on any one unit. The strip alloying process allows precise monitoring of the alloying conditions of each transistor. Because of the precision alloying made possible by the new technique, greater built-in reliability is assured.

The fabrication of SPAT transistors has not necessitated usage of any new or different materials, nor has there been any need for device design changes which otherwise might have affected behavior of the units on extended life.

Eight types, all in TO-18 enclosures, are listed in the new SPAT line which Philco put in production in January, 1961.

Large Man-Made Diamonds.—For the first time in history, man has made large diamonds, over a carat in size. They are dark in color and, because of structural imperfections, they are not yet of sufficient mechanical strength for industrial application.

This scientific feat was achieved at the General Electric Research Laboratory in Schenectady, where the first man-made diamonds were announced in 1955. These new, large diamonds are the latest milestone in a high-temperature, high-pressure research program that GE has pursued since its initial success in converting common graphite into diamonds. The large diamond work is still in the early development stage, and it is impossible to predict when the Company will succeed in making large diamonds with mechanical properties comparable to the small man-made diamonds, which have proved to be superior to natural diamonds for many industrial applications. Industrial diamonds are of the utmost strategic importance today, for the major source of natural diamonds is the Congo, and the uncertain conditions there demonstrate how fortunate it is that industry can turn to man-made diamonds for many of its needs.

Industry uses diamonds in full carat ranges for drills, dressing tools, dies, and single point cutting tools. If General Electric can perfect the mechanical structure and improve the strength of its new, carat-size diamonds, the Company will be able to compete with natural stones in the full industrial range.

Stratoscope II.—Stratoscope II, an attempt to photograph planets and stellar nebulae with clarity never before possible, is in the preliminary testing stage, according to a joint announcement from the National Science Foundation, the Navy, and Princeton University.

Heart of the project is a balloon-borne 30-in. telescope system weighing more than two tons, which will be lofted to 80,000 ft. (more than 15 miles), where it will be above nearly all of the atmospheric turbulence and dust that distorts or obscures ground-based observations. This permits photographs to be made with a resolution greater than from the ground by a factor of three to five.

The project is under the direction of Prof. Martin Schwarzschild, internationally noted astronomer at Princeton University, who also directed the highly successful flights of a 12-in. solar telescope, Stratoscope I, in 1957 and 1959. The new program is sponsored by the National Science Foundation and the Office of Naval Research, with additional support from the National Aeronautics and Space Administration.

The balloon will remain aloft throughout the night while observations are being made. Among the first objectives of the Princeton astronomers are:

—Analysis of the divisions in Saturn's rings, possibly throwing more light on the factors that determine the orbits of the many particles that make up the rings.

—Studies of the sudden atmospheric changes that take place on Jupiter,

which may help to explain Jupiter's mysterious Red Spot, and on Venus, and of the possible relationships they may have to solar magnetic storms.

—Close examination of the gaseous nebulae between stars (from which new stars are formed), providing possible clues to the way our sun was formed.

—If conditions are right, the possibility of achieving man's first look at the surface of cloud-shrouded Venus through brief holes in the cloud cover.

A proposed balloon system to lift the telescope payload, which weighs about 4300 pounds including associated electronic equipment, is under development. The balloon will use a new material known as S-10, which is basically Mylar plastic reinforced with a dacron mesh and is lighter and stronger than polyethylene. A method of utilizing a helicopter to capture the balloon and its payload as it descends, in order to achieve a soft landing, is also under development.

An initial test of a helicopter-towed device for snagging the balloon has been completed by the Naval Air Development Test Unit, So. Weymouth, Mass. The first test launch of the new system with a dummy payload and also of the helicopter retrieval method is scheduled for early 1961. The target period for the first full-scale launch with the telescope is fall, 1961.

The telescope itself is an L-shaped structure, with one arm measuring about 25 ft. and the other nearly 19 ft. This unique instrument will have a theoretical resolving power of $\frac{1}{10}$ of a second of arc, or the ability to distinguish two golf balls 15 inches apart 500 miles away.

Corning Glass Works, Corning, N. Y., has fabricated the 36-inch quartz optical mirror for the telescope. This is the first time a quartz blank of this size has been successfully cast. Quartz is used because it is unaffected by the sharp changes in temperature such as Stratoscope II will experience. The mirror has been ground with unusually high accuracy to a true parabolic shape with a tolerance of one-millionth of an inch.

While the telescope is at altitude, it will be focused and monitored by means of a remote control television system and a 70-station command channel developed by RCA Laboratories, Princeton, N. J. This TV system, utilizing RCA's new high resolution image orthicon tube, will be further developed for the satellite telescope observatory planned by NASA. A telemetry system will be provided by the Sierra Research Laboratory, Buffalo, N. Y.

New Chemical Additive for Jet Fuel Systems.—A new chemical additive which prevents in-flight ice formation in aircraft jet fuel systems has been developed by Phillips Petroleum Company of Bartlesville, Oklahoma. It has successfully undergone exhaustive tests by the United States Air Force and the Boeing Airplane Company. The Air Force has approved its use by the Strategic Air Command.

Phillips developed the new compound, called PFA 55MB, with its own research funds. It is immediately available in sufficient quantities to meet the needs of military and commercial jet aircraft and guided missiles using jet fuel.

The new additive was chosen after elimination of hundreds of other candidates which were screened in extensive laboratory work. PFA 55MB has been proved satisfactory and effective in a year of flight tests in a B-52G at Boeing-Wichita and thousands of hours of tests in 15 B-52s and 10 KC-135 jet tankers of the

4043rd Strategic Wing, Wright-Patterson Air Force Base. The new additive is compatible with materials it contacts in fuel systems. It does not adversely affect the thermal stability of jet fuel.

High-altitude jet aircraft constantly fly in sub-zero temperatures. In the event that the fuel system contains any water, from condensation or any other source, it may freeze into ice. This can block fuel flow and cause loss of engine power or flame-out.

Some jet aircraft now rely on heaters installed between the fuel tanks and the turbines to raise in-flight fuel temperatures to safe limits. A very small amount of the anti-icing compound, added to jet fuel, provides more complete protection than heaters. This patented development also prevents ice formation from condensation and other sources in the giant fuel tanks and other parts of the aircraft fuel system not served by heaters.

In new aircraft, use of the additive could eliminate the initial cost, weight, maintenance and in-flight operating expense of heaters. Even in aircraft now equipped with heaters, it appears to offer economic advantages.

Slide Rule Plots Orbiting Satellites.—A new type of slide rule which quickly gives the location of a satellite moving around the Earth was recently completed for use by the Air Research and Development Command (ARDC).

Developed for ARDC's Rome Air Development Center by Planning Research Corporation, Los Angeles, Calif., and called the Satellite Trajectory Simulator, the computing device pinpoints the path of a satellite and determines geographical areas visible from the satellite.

In addition, the simulator gives the frequency and times it will pass over any particular predetermined points on the ground. The slide rule also may be worked backwards to determine the launch conditions (time and place) necessary to achieve a satellite's journey.

The simulator, to be used as a visual aid and quick reference device, consists of a square metallic base and two replaceable concentric, transparent plastic disks fixed to the base at the center with free independent rotation about the center.

The slide rule can be made in any convenient size, depending largely on the accuracy needed, commercial materials available, and ease of handling.

Planning Research Corporation developed the device under contract with the Intelligence Laboratory of ARDC.

Transparent Coating for Metals.—A newly developed perspiration resistant, water clear transparent coating for application on brass, copper, and aluminum, has just been announced by the Sun Chemical Corporation.

Clear "Sunflex" is also very resistant to marring, scuffing, bruising, scratching and abrasion, and therefore recommended for lipstick cases, compacts, hardware and drawer pulls, door knobs, escutcheon plates, etc., all of which are called upon constantly to endure handling and normal abuse. Lengthy, rigorous, simulated perspiration tests and accelerated scuff, bruise, scratch and abrasion tests were conducted prior to the introduction of this product to the trade, and before this announcement.

This material is basically a highly specialized synthetic resin with an unusually fast curing ability. It may be applied by dipping or by spraying, either with conventional spray equipment or with electrostatic spray machines. Its long wearing, hard,

durable, resistant finish is fast cured after baking for only two minutes at 400°F.

"Sunflex" is an entirely new product of the Chemical Coatings Division of Sun, and was developed to fill the long desired need for a compound serving this purpose.

Ceramic Insulation.—New ceramic insulations capable of withstanding red heat temperatures of over 500°C. have been successfully developed by the General Electric Company.

The new insulation materials will be used in electrical systems for Mach 3 aircraft now under development, as well as for future missiles and other space applications.

Intense temperatures of Mach 3 operation, expected to range around 500°C., and in hot spots up to 600°, narrowed the field of available materials to ceramics and other inorganic materials. The best organic or metal-organic polymers have a short life above 300°C.

Because generators require two different types of insulation—one for ground and phase insulation and the other for an encapsulating coating—two types of inorganic insulation materials were developed.

The most successful solution to the ground and phase problem was an inorganic sheet material consisting of a composite of flake mica and glass cloth bonded together with an inorganic cement.

For the encapsulating material, a ceramic cement was developed. A successful formulation consisted of equal parts of magnesium oxide and glass frit, with a binder of aluminum phosphate in phosphoric acid.

Although most ceramic cements have short pot lives, which makes processing difficult, the cement developed by General Electric had a pot life in excess of 30 minutes. In addition, it showed less than 1 per cent moisture absorption after 2-hr. immersion in boiling water.

Testing of scores of ceramic materials began two years ago with the specific goal of developing insulation for 40 kva generators under the Air Force's "Hotelec" program, managed by North American. The completed insulation system was installed in 40 kva generators and successfully tested under aircraft environmental conditions.

In laboratory tests, the ceramic materials withstood 40-hr. operation at 500°C. and remained in good operating condition after many additional hours of testing at 300 to 500°.

Plastic Foam Suitable in Space Environment.—Man may be able to "set up light housekeeping" within minutes after he lands on other planets thanks to a lightweight plastic foam.

Air Research and Development Command's studies of the foam, a polyurethane substance, have disclosed its potential structural applications in space environments.

Man, arriving on another planet with a compact, lightweight kit, could erect a shelter made of the foam. He could also fabricate internal furnishings in a matter of minutes, eliminating the need to transport bulky desks, beds, chairs, laboratory benches and storage compartments.

While the foam alone would probably be unable to stand the pressure of sitting, sleeping, and work loads on Earth, it would hold up well in the reduced gravity of space.

In one method used in the WADD experiments, a polyethylene jar containing the prepolymer resin was placed within a medium-size laboratory bell jar. The jar was evacuated

to 3.5 mm. mercury and a catalyst, trimethylamine adsorbed in a silica gel, was added through a tube into the prepolymer resin. The materials then were stirred for about 30 seconds —until a foam reaction began.

Water, necessary for carbon dioxide generation, was also adsorbed on the silica gel. The advantage of the silica gel catalyst mixture technique is that it releases the necessary ingredients at a controlled rate. Experiments showed that if the carbon dioxide was generated at too rapid a rate, the foam would collapse before the plastic could cure sufficiently to maintain a cellular structure.

Efficient operation in producing the foam at low pressure environments could make it possible not only for man to build plastic "space shelters" and equipment in a short time, but also would provide a means of rigidifying weather and communications satellite balloons, such as Echo I.

In addition, a comparatively thin layer of foam applied to the exterior of a long-term orbiting manned satellite, would be one means of absorbing the energy of meteorite particles. It also would be possible to color the foam to achieve a measure of temperature control.

Plasma Diode.—General Electric scientists are testing a promising thermionic conversion device for use in nuclear reactors in the next 5 to 10 years.

The device, known as a plasma diode, utilizes the tremendous heat produced in the center of nuclear reactor fuel elements for the direct generation of electricity without the necessity for mechanical rotating equipment.

A thermionic converter produces an electrical current by boiling electrons off a cathode, or hot metal plate, and collecting them on an anode, a cooler plate.

A company spokesman said refinement of the present experimental diodes would permit a sizeable increase in the electrical output of nuclear power stations at slight additional cost.

He added that application of such devices to conventional steam generating stations does not appear practical, since the normal temperature levels achieved in such stations are too low to permit efficient operation of such direct conversion devices.

The plasma diode under development is a high-temperature thermionic device using cesium gas and a uranium-bearing cathode.

These diodes would be built into nuclear fuel elements, where the fissioning of the uranium in the cathode would provide the high temperatures —in the range of 4,500° F.—needed for efficient direct conversion. An electrical lead from the fuel element would draw off the electricity.

In addition to the directly produced electricity, lower temperature heat rejected through the walls of the fuel element would also be used to produce electricity by boiling water to make steam for the operation of turbine generator facilities.

The plasma diode project is being conducted by General Electric's Vallecitos Atomic Laboratory here, nuclear research center of the company's Atomic Power Equipment Department, headquartered in San Jose, Calif. Also cooperating in the project are the company's Research Laboratory and General Engineering Laboratory, both in Schenectady, N. Y.

The Vallecitos laboratory has conducted various exploratory studies on direct conversion over the last 18 months, and the promising results obtained in studies of high temperature vapor thermionic devices led to the

decision to expand and intensify efforts in this area.

Developmental diodes are now being made at Vallecitos, and single-cell devices able to generate several watts are currently undergoing tests in a special fuel capsule in the General Electric Test Reactor here.

Present plans call for additional capsule tests in the continuing program to gain more experimental data.

Although thermionic conversion devices capable of producing approximately 20 watts per square centimeter of cathode area have been demonstrated, present thermionic converters are still a long way from approaching practical application.

Recent design advances by Vallecitos project scientists, however, lead them to believe they can have an advanced plasma diode ready to be installed in a thermionic fuel element and tested in a prototype nuclear power plant within five years.

Operation of the prototype plant would provide the data and experience necessary for the company to design large central station nuclear plants of this type for construction in 1970.

Manager of the new project is H. Malcolm Ogle, holder of 24 U. S. patents and a veteran of 19 years with General Electric, the last five with the Atomic Power Equipment Department.